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RESEARCH ARTICLE

INTEGRAL SOLUTION OF THE TERNARY CUBIC EQUATION $6(x^2+y^2)-11xy+x+y+1=552z^3$

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ABSTRACT

The ternary cubic condition $6(x^2+y^2)-11xy+x+y+1=552z^3$ is considered for deciding its non-zero particular basic arrangements utilizing the linear transformations $x=u+v, y=u-v\ (u\neq v\neq 0)$, and utilizing the strategy for factorization in complex forms, various methods of essential answers for the ternary cubic condition viable are gotten. In each method, fascinating relations among the arrangements, a few exceptional polygonal, pyramidal numbers and focal pyramidal numbers are displayed.

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INTRODUCTION

The subject of Diophantine condition, one of the fascinating areas of Number Theory, assumes a critical part in higher math and greatly affects gullible individuals and consistently possesses a surprising situation because of unchallenged verifiable significance. The Diophantine conditions might be either polynomial condition with no less than two questions for which number arrangement, are required or supernatural condition including geometrical, logarithmic, outstanding and surd capability to such an extent that one might be keen on getting whole number arrangement.

The issue of tracking down all number arrangements of a Diophantine condition with at least three factors and conditions of degree something like three, in everyday presents a reasonable plan of challenges. In [1-3], hypothesis of numbers was talked about. In [4,5], an exceptional Pythagorean triangle issue have been examined for its essential arrangements.

In [6-11], higher request conditions are considered for essential arrangements. In this correspondence, the non-homogeneous cubic condition with three unknowns addressed by the condition $6(x^2+y^2)-11xy+x+y+1=552z^3$ is thought of and specifically a couple of fascinating relations among the arrangements are introduced.

METHOD OF ANALYSIS

The ternary cubic equation to be solved is

$$6(x^2+y^2)-11xy+x+y+1=552z^3 \qquad \text{……………..} (1)$$

Introducing the linear transformations

$$x=u+v \quad ; \quad y=u-v \qquad \text{……………..} (2)$$

in (1), it is written as

$$(u+1)^2+23v^2=552z^3 \qquad \text{……………. (3)}$$

Now (3) is solved through different ways and using (2), different methods of integer solutions to (1) are obtained.

Method 1

Assume, $z=a^2+23b^2$ ……………. (4)

Write 552 as,

$$552=\left(23+i\sqrt{23}\right)\left(23-i\sqrt{23}\right) \qquad \text{……………. (5)}$$

Using (4) and (5) in (3) and applying the method of factorization, define

$$(u+1)+i\sqrt{23}\,v=\left(23+i\sqrt{23}\right)\left(a+i\sqrt{23}b\right)^3 \qquad \text{……………. (6)}$$

Equating the real and imaginary parts, the values of u and v are obtained as

$$u+1=23a^3-1587ab^2-69a^2b+529b^3$$

$$v=a^3+69a^2b-69ab^2-529b^3$$

Substituting the above values of u and v in (2), the values of x and y are given by

$$x=24a^3-1656ab^2-1$$

$$y=22a^3-1518ab^2-138a^2b+1058b^3-1$$

$$z=a^2+23b^2$$

Properties:

1. $y(a,a)+288SO_a+144Gno_a\equiv 0(\text{mod }145)$
2. $z(a,a)-12CS_a-12Gno_a\equiv 0$
3. $816T_{3,a}-x(a,a)-816P_a^5-816Gno_a\equiv 0(\text{mod }817)$
4. $x(a,a)+816SO_a+408Gno_a\equiv 0(\text{mod}409)$
5. $y(a,a)-2x(a,a)-1344SO_a-672Gno_a\equiv 0(\text{mod }673)$
6. $2y(a,a)-x(a,a)-240SO_a-120Gno_a\equiv 0(\text{mod}119)$
7. $x(a,a)-2y(a,a)-z(a,a)+240P_a^5+228T_{6,a}+114Gno_a\equiv 0(\text{mod}115)$

Method 2

Equation (3) can be written as

$$(u+1)^2+23v^2=552z^3$$

Write $552=\dfrac{\left(19+i4\sqrt{23}\right)\left(19-i4\sqrt{23}\right)}{27^2}$ …………….(7)

Define,

$$(u+1)+i\sqrt{23}v=\frac{\left(19+i4\sqrt{23}\right)\left(23+i4\sqrt{23}\right)\left(a+i\sqrt{23}b\right)^3}{27}$$

Equating real and imaginary parts, leads to

$$u+1=\frac{1}{27}[345a^3-23805ab^2-7659a^2b+58719b^3] \qquad \text{…………….(8)}$$

$$v=\frac{1}{27}[111a^3-7935b^3+1035a^2b-7659ab^2] \qquad \text{…………….(9)}$$

Since our aim is to find integer solution, assuming $a=27A$ and $a=27B$ in (8) and (9) and substituting the values of u, v in (2), the distinct nonzero integral solutions to (1) obtained as

$x = 27^2[456A^3 - 31464AB^2 - 6624A^2B + 50784B^3] - 1$

$y = 27^2[234A^3 - 16146AB^2 - 8694A^2B + 66654B^3] - 1$

$z = 27^2[A^2 + 23B^2]$

Properties:

1. $z(a,1) - 243T_{8,a} - 243Gno_a \equiv 0(\bmod 17010)$
2. $z(1,a) - 1863T_{20,a} - 7452Gno_a \equiv 0(\bmod 8181)$
3. $2z(a,1) - 243T_{14,a} - T_{6,a} + CS_a - 607Gno_a \equiv 0(\bmod 34142)$
4. $x(a,a) - 4793904SO_a - 2396952Gno_a \equiv 0(\bmod 2396951)$
5. $y(a,a) - 15326496P_a^5 + 15326496T_{3,a} - 15326496Gno_a \equiv 0(\bmod 15326495)$
6. $x(a,1) - 20776TO_a - 4CC_a + 2071638T_{3,a} + 9646308Gno_a \equiv 0(\bmod 27499887)$

Method 3

Equation (3) can be written as

$(u+1)^2 + 23v^2 = 552z^3 * 1$

Write '1' as, $1 = \frac{(22 + i2\sqrt{23})(22 - i2\sqrt{23})}{24^2}$

Define, $(u+1) + i\sqrt{23}v = \frac{(23 + i\sqrt{23})(22 - i2\sqrt{23})(a + i\sqrt{23}b)^3}{24}$

Equating real and imaginary parts,

$$u + 1 = \frac{1}{24}[460a^3 - 31740ab^2 - 4692a^2b + 35972b^3] \quad \text{……………..(10)}$$

$$v = \frac{1}{24}[68a^3 + 1380a^2b - 4692ab^2 - 10580b^3] \quad \text{……………..(11)}$$

Since our aim is to find integer solutions, assuming $a = 24A, b = 24B$ in (10) and (11) and substituting the values of u, v in (2), the distinct non-zero integral solutions to (1) determined as

$x = 24^2[528A^3 - 36432AB^2 - 3312A^2B + 25392B^3] - 1$
$y = 24^2[392A^3 - 27048AB^2 - 6072A^2B + 46552B^3] - 1$
$z = 24^2[A^2 + 23B^2]$

Properties:

1. $x(a,a) + y(a,a) + z(a,a) - 768T_{11,a} - 5376Gno_a \equiv 0(\bmod 5374)$
2. $6912T_{3,a} - z(a,a) - 3456Gno_a \equiv 0(\bmod 3456)$
3. $x(1,1) + y(1,1) + z(1,1) + 2$ is a cubical integer.
4. $\frac{1}{24^2}[y(a,a)] + 1$ is a cubical integer.
5. $\frac{1}{24^2}[y(a,a) - z(a,a)] - 3456RD_a - 1726T_{26,a} - 2581Gno_a \equiv 0(\bmod 6036)$
6. $\frac{1}{24^2}[x(a,1) - z(a,1)] - 264CC_a + T_{8,a} + 1259T_{3,a} + 17352Gno_a \equiv 0(\bmod 8280)$

Method 4

Equation (3) can be written as

$(u+1)^2 + 23v^2 = 552z^3 * 1$

Write '1' as, $1 = \frac{(13 + i7\sqrt{23})(13 - i7\sqrt{23})}{36^2}$

Define, $(u+1)+i\sqrt{23}\,v=\dfrac{(23+i\sqrt{23})(13+i7\sqrt{23})(a-i\sqrt{23}\,b)^3}{36^2}$

Equating real and imaginary parts,

$$u+1=\frac{1}{36}[138a^3-9522ab^2-12006a^2b+92046b^3] \quad \text{……………..(12)}$$

$$v=\frac{1}{36}[174a^3+414a^2b-12006ab^2-3174b^3] \quad \text{……………..(13)}$$

Since our aim is to find integer solution, assuming $a=36A$ and $b=36B$ in (12) and (13) and substituting the values of u,v in (2), the distinct nonzero integral solutions to (1) found as

$$x=36^2[312A^3-21528AB^2-11592A^2B+88872B^3]-1$$

$$y=36^2[-36A^3+2484AB^2-12420A^2B+95220B^3]-1$$

$$z=36^2[A^2+23B^2]$$

Properties:

1. $x(a,1)-202176\,CC_a+7208352\,T_{3,a}+7045056\,Gno_a\equiv 0 \pmod{108335231}$
2. $z(4,b)-9936T_{8,a}-9936Gno_a\equiv 0 \pmod{30672}$
3. $18413568\,O_a-y(a,a)-27620352\,Gno_a\equiv 0 \pmod{27620353}$
4. $x(a,a)-z(a,a)-5184RD_a-36319104SO_a-18149184Gno_a\equiv 0 \pmod{18154367}$
5. $\frac{1}{36^2}[x(a,a)-z(a,a)]-28032\,CC_a-10509\,T_{18,a}\equiv 10533 \pmod{28031}$
6. $\frac{1}{36^2}9z(a,a)$ is a cubical integer.
7. $\frac{1}{36^2}6z(a,a)$ is a perfect square.

CONCLUSION

In this paper, we have presented four different methods of non-zero distinct integer solutions of the non homogeneous cone given by $6(x^2+y^2)-11xy+x+y+1=552z^3$ is presented to conclude, one may search for other patterns of non-zero distinct integer solutions and their corresponding properties for other choices of cubic Diophantine equations.

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