Available Online at http://www.journalajst.com

ASIAN JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN: 0976-3376

Asian Journal of Science and Technology
Vol. 16, Issue, 03, pp. 13559-13561, March, 2025

RESEARCH ARTICLE

COMMON COINCIDENCE POINTS FOR TWO PAIRS OF R-WEAKLY COMMUTATIVE MAPPINGS ON B2-MULTIPLICATIVE METRIC SPACE

***Manish Kumar**

(Department of Mathematics), (Government Degree College Nanauta, Saharanpur)

ARTICLE INFO

Article History:
Received 08th January, 2025
Received in revised form
29th January, 2025
Accepted 03rd February, 2025
Published online 24th March, 2025

Keywords:
B metric Space, 2-metric space, multiplicative metric space, B2-multiplicative metric space, R-weakly commutative mappings, Coincidence point.

ABSTRACT

In [22], Kumar introduced the notion of b2-multiplicative metric space. Czerwik [2] generalized the concept of metric space and put the idea and terminology of b-metric space. In [15] B. Surender Reddy et al presented the concept of 2-multiplicative metric space. In this paper we introduce the concept of R-weakly commutative mappings on b2-multiplicative metric space. Also we prove a common coincidence point theorem for two pairs of R-weakly commutative mappings on b2-multiplicative metric space.

Citation: ***Manish Kumar.* 2025.** "Common coincidence points for two pairs of r-weakly commutative mappings on b2-multiplicative metric space", *Asian Journal of Science and Technology*, 16, (03), 13559-13561.

INTRODUCTION

Grossman and Katz [11] introduced a new kind of Calculus called multiplicative (or non-Newtonian) calculus by interchanging the roles of subtraction and addition with the role of division and multiplication, respectively. By using the ideas of Grossman and Katz [11], Bashirov et al. [7] defined the notion of multiplicative metric. Czerwik [2] introduced the notion of b-metric space which is a generalization of a metric space. There are some fixed point theorems in b-metric spaces. Huang et al. [14] introduced fixed point results for rational Geraghty contractive mappings; Ozturk and Turkoglu [20] studied fixed points for generalized alpha-psi-contractions; Shatanawi et al. [22] established a study of contraction conditions using comparison functions. The notion of a 2-metric space was introduced by Gahler, in [4]. Several fixed-point results were obtained in [1,2,3,4,5 6], as a generalization of the concept of a metric space. A 2-metric is not acontinuous function of its variables, whereas an ordinary metric is. The basic philosophy is that since a 2-metric measures area, a contraction should send the space towards a configuration ofzero area, which is to say a line. Z. Mustafa introduced a new type of generalized metric space called b2-metric space, as a generalization of the 2-metric space, [8]. In[15] B. Surender Reddy et al presented the conception of 2-multiplicative metric space and 2-multiplicative Normed linear space and investigate topological properties in 2-multiplicative NDLS. In [22], Kumar introduced the notion of b2-multiplicative metric space. The aim of this paper is to introduce the concept of R-weakly commutative mappings on b2-multiplicative metric and then we prove a common coincidence point result for two pairs of R-weakly commutative mappings on b2-multiplicative metric space.

Preliminaries

Definition 1.1. [4, 9] Let X be a non-empty set and $d : X\times X\times X\to R_+$ be a map satisfying the following properties

(i) $d(x,y,z) = 0$ if at least two of the three points are the same.
(ii) For $x,y\in X$ such that $x \neq y$ there exists a point $z \in X$ such that $d(x,y,z)\neq 0$.
(iii) symmetry property: for $x,y,z\in X$, $d(x,y,z) = d(x,z,y) = d(y,x,z) = d(y,z,x) = d(z,x,y) = d(z,y,x)$.
(iv) rectangle inequality: $d(x,y,z)\leq d(x,y,t) + d(y,z,t) + d(z,x,t)$ for $x,y,z,t\in X$.

Then d is a 2-metric and (X,d)is a 2-metric space.

Definition 1.2. [8] Let X be a non-empty set and $d : X\times X\times X\to R_+$ be a map satisfying the following properties

(i) $d(x,y,z) = 0$ if at least two of the three points are the same.
(ii) For $x,y\in X$ such that $x \neq y$ there exists a point $z \in X$ such that $d(x,y,z)\neq 0$.
(iii) symmetry property: for $x,y,z\in X$, $d(x,y,z) = d(x,z,y) = d(y,x,z) = d(y,z,x) = d(z,x,y) = d(z,y,x)$.
(iv) s-rectangle inequality:there exists $s \geq 1$ such that $d(x,y,z)\leq s[d(x,y,t) + d(y,z,t) + d(z,x,t)]$ for $x,y,z,t\in X$.

Then d is a b2-metric and (X,d)is a b2-metric space
If s=1, the b2-metric reduces to the 2-metric.

Definition 1.3. [10] Let X be a non-empty set and $d : X\times X\times X\to R_+$ be a map satisfying the following properties:

(i) d(x,y,z) = 0if at least two of the three points are the same.
(ii) For x,y∈X such that x ≠y there exists a point z ∈X such that d(x,y,z)≠0.
(iii) symmetry property: for x,y,z∈X, d(x,y,z) = d(x,z,y) = d(y,x,z) = d(y,z,x) = d(z,x,y) = d(z,y,x).
(iv) modified rectangle inequality:there exists α,β,γ≥1such that d(x,y,z)≤αd(x,y,t) + βd(y,z,t) + γd(z,x,t)] for x,y,z,t∈X.

Then d is a generalized b2-metric and (X,d)is a generalized b2- metric space.

Definition 1.4.[16] Let X be a nonempty set and let s ≥ 1 be a given real number. A mapping m: X × X → [1, ∞) is called a b-multiplicative metric if the following conditions hold:

(m1) m(x, y) > 1 for all x, y ∈ X with x≠ y and m(x, y) = 1 if and only if x = y;
(m2) m(x, y) = m(y, x) for all x, y ∈ X;
(m3) m(x, z) ≤ m(x, y)^s · m(y, z)^s for all x, y, z ∈ X.

The triplet (X, m, s) is called a b-multiplicative metric space.

Definition 3.1[15]: A product 2-metric on X is a mapping ¯h : X × X × X → R+ that satisfies the following conditions.

(i) h(ϑ1, ϑ2, ϑ3) ≥ 1, ∀ ϑ1, ϑ2, ϑ3 ∈ X and h(ϑ1, ϑ2, ϑ3) = 1 when two of the three elements ϑ1, ϑ2, ϑ3 ∈ X are equal
(ii) h(ϑ1, ϑ2, ϑ3) = h(ϑ1, ϑ3, ϑ2) = h(ϑ2, ϑ1, ϑ3) = ... ∀ ϑ1, ϑ2, ϑ3 ∈ X
(iii) h(ϑ1, ϑ2, ϑ3) ≤ h(ϑ1, ϑ2, a).h(ϑ1, a, ν3).h(a, ϑ2, ϑ3) ∀ ϑ1, ϑ2, ϑ3, a ∈ X.

The pair (X, h) is known as a product 2-MCS.

Definition 3.1[22]: Let X be a nonempty set and let s ≥ 1 be a given real number. A mapping m: X × X × X → [1, ∞) is called a b2-multiplicative metric if the following conditions hold:

(m1) m(x, y, z) > 1 for all x, y, z∈ X and m(x, y, z) = 1 when two of the three elements x. y.

z∈ X are equal;
(m2) m(x, y, z) = m(p(x, y, z)) for all x, y, z∈ X and p(x, y, z) is any permutation of x, y, z;
(m3) m(x, y, z) ≤ m(x, y, a)^s · m(x, a, z)^s· m(a, y, z)^{s}for all x, y, z, a∈ X.

The triplet (X, m, s) is called a b2-multiplicative metric space.

Example 2.1[22]. Let X = [0, ∞). Define a mapping m_a: X × X× X → [1,∞), $m_a(x, y, z) = a^{\min\{(x-y)^2,(y-z)^2,(z-x)^2\}}$, where a > 1 is any fixed real number. Then for each a, m_a is b2-multiplicative metric on X with s = 2. Note that ma is not a 2-multiplicative metric on X.

Let (X, m, s) is a b2-multiplicative metric space. Then the multiplicative open and closed 2-ball of radius ε > 1 having center at x and y is of the form: $B_ε(x, y)$ = {a∈ X : m(x, y, a) < ε} and $B_ε(x, y)$ = {a∈ X : m(x, y, a) ≤ ε} respectively.

Let {x_n} be a sequence in a b2-multiplicative metric space (X, m, s).

1. {x_n} is said to be b2-multiplicative convergent to x∈X, written as $\lim_n x_n$=x, if for alla∈ X $\lim_n m(x_n,x,a)$=0.
2. {x_n} is said to be a b2-Cauchy sequence in *X* if for all a∈X, $\lim_{n,m} d(x_n,x_m,a)$=0.
3. (X, m, s) is said to be b2-complete if every b2-Cauchy sequence is a b2-convergent sequence.

RESULTS

Definition3.1: Let (X, m, s) be a b2-multiplicative metric space. Two mappingsA, B : X → X are said to be R-weakly commutative if
m(ABx, BAx, a)≤m(Ax, Bx, a)
for all x, a ∈X.

Theorem 3.1. Let (X, m, s) be a complete b2-multiplicative metric space. Let A, B, S, T : X → X be mappings such that for each x, y, a∈X,

(i) AX ⊆ TX and BX ⊆ SX;
(ii) The pairs {A, S} and {B, T} are R-weakly commutative;
(iii) $m(Ax, By, a) \le \max\{m(Sx, Ty, a), m(Sx, Ax, a), m(Ty, By, a), m(Sx, By, a)^{1/2s} \cdot m(Ty, Ax, a)\}^{\kappa}$

where κ ∈ [0, 1/s).

Then the pairs {A, S} and {B, T} has acommon coincidence point in X.

Proof. As we have x_0∈ X such that $(x_0, x_1, a) \in E_a$, where $y_0= Ax_0= Tx_1$andx_2∈ Xs. t. $y_1= Bx_1= Sx_2$ and thus we get sequences {x_n} and {y_n} such that

$y_{2n}= Ax_{2n} = Tx_{2n+1}$and $y_{2n+1}= Bx_{2n+1}= Sx_{2n+2}$n = 0, 1, 2, 3, …….
From (1), we have
$m(y_{2n}, y_{2n+1}, a) = m(Ax_{2n}, Bx_{2n+1}, a)$
$\le \max\{m(Sx_{2n}, Tx_{2n+1}, a), m(Sx_{2n}, Ax_{2n}, a), m(Tx_{2n+1}, Bx_{2n+1}, a), m(Sx_{2n}, Bx_{2n+1}, a)^{1/2s} \cdot m(Tx_{2n+1}, Ax_{2n}, a)\}^{\kappa}$
$= \max\{m(y_{2n-1}, y_{2n}, a), m(y_{2n-1}, y_{2n}, a), m(y_{2n}, y_{2n+1}, a), m(y_{2n-1}, y_{2n+1}, a)^{1/2s} \cdot m(y_{2n}, y_{2n}, a)\}^{\kappa}$
$= \max\{m(y_{2n-1}, y_{2n}, a), m(y_{2n-1}, y_{2n}, a), m(y_{2n}, y_{2n+1}, a)\}^{\kappa}$
$= m(y_{2n-1}, y_{2n}, a)^{\kappa}$.
Also, we have
$m(y_{2n+1}, y_{2n+2}, a) = m(Bx_{2n+1}, Ax_{2n+2}, a)= m(Ax_{2n+2}, Bx_{2n+1}, a)$
$\le \max\{m(Sx_{2n+2}, Tx_{2n+1}, a), m(Sx_{2n+2}, Ax_{2n+2}, a), m(Tx_{2n+1}, Bx_{2n+1}, a), m(Sx_{2n+2}, Bx_{2n+1}, a)^{1/2s} \cdot m(Tx_{2n+1}, Ax_{2n+2}, a)\}^{\kappa}$
$= \max\{m(y_{2n+1}, y_{2n}, a), m(y_{2n+1}, y_{2n+2}, a), m(y_{2n}, y_{2n+1}, a), m(y_{2n+1}, y_{2n+1}, a)^{1/2s} \cdot m(y_{2n}, y_{2n+2}, a)^{1/2s}\}^{\kappa}$
$= \max\{m(y_{2n}, y_{2n+1}, a), m(y_{2n+1}, y_{2n+2}, a), m(y_{2n}, y_{2n+1}, a)\}^{\kappa}$
$= m(y_{2n}, y_{2n+1}, a)^{\kappa}$.
So in general we get
$m(y_n, y_{n+1}, a) \le m(y_{n-1}, y_n, a)^k$(2)
repeating application of (2) yields
$m(y_n, y_{n+1}, a) \le m(y_0, y_1, a)^{k^n}$

Continuing in the same way, we construct a sequence {x_n} in X such that
$x_{n+1} = fx_n$, $(x_n, x_{n+1}, a) \in E$ and $m(x_n, x_{n+1}, a) \le m(x_0, x_1, a)^{k^n}$ for each n ∈ N.

Let m, n ∈ N, then by themultiplicative triangular inequality, we get

$m(y_n, y_{n+m}, a) \le m(y_n, y_{n+1}, a)^{s^n} . m(y_{n+1}, y_{n+2}, a)^{s^{n+1}} \dots\dots\dots . m(y_{n+m-1}, y_{n+m}, a)^{s^{n+m}}$
$\le m(y_0, y_1, a)^{(ks)^n} . m(y_0, y_1, a)^{(ks)^{n+1}} \dots\dots\dots . m(y_0, y_1, a)^{(ks)^{n+m}}$
$\le m(y_0, y_1, a)^{\frac{(ks)^n}{1-ks}}$

Letting n → ∞, in above inequality, we get $m(y_n, y_{n+m}, a) \to_{b2} 1$. Hence the sequence {$y_n$}, and hence it's sub-sequence is 2-multiplicative Cauchy sequence. By the completeness of T(X), $Ax_{2n}= Tx_{2n+1} \to_{b2} y$ = Tx for some x∈ X.

$m(Sx_{2n}, Tx, a) \le m(Sx_{2n}, Tx_{2n+1}, a)^s . m(Tx_{2n+1}, Tx, a)^s . m(Sx_{2n}, Tx, Tx_{2n+1})^s \to 1$ as n →∞.
Hence $Sx_{2n} \to Tx$ as n →∞.

Now we have

$m(y, Bx, a) \leq m(y, Ax_{2n}, a)^s \cdot m(y, Bx, Ax_{2n})^s \cdot m(Ax_{2n}, Bx, a)^s$
$\leq m(y, Ax_{2n}, a)^s \cdot m(y, Bx, Ax_{2n})^s \cdot \max\{m(Sx_{2n}, Tx, a), m(Sx_{2n}, Ax_{2n}, a),$
$m(Tx, Bx, a), m(Sx_{2n}, Bx, a)^{1/2s} \cdot m(Tx, Ax_{2n}, a)\}^{\kappa s}$
$= 1.1.\max\{1, 1, m(y, Bx, a)\}^{ks}$

which gives $m(y, Bx, a) = 1$ otherwise we get a contradiction $m(y, Bx, a) \leq m(y, Bx, a)^{ks}$. So we get $Bx = y = Tx$. Since $BX \subseteq SX$, there exists an element $z \in X$ such that $Tx = Bx = Sz$.

We have

$m(Az, Sz, a) = m(Az, Bx, a) \leq \max\{m(Sz, Tx, a), m(Sz, Az, a), m(Tx, Bx, a),$
$m(Sz, Bx, a)^{1/2s} \cdot m(Tx, Az, a)\}^{\kappa}$
$= \max\{1, m(Az, Sz, a), 1, m(Az, Sz, a)\}^k$

which gives $m(Az, Sz, a) = 1$ otherwise we get a contradiction $m(Az, Sz, a) \leq m(Az, Sz, a)^k$.

So we obtain $Az = Sz$. Thus $Az = Sz = Tx = Bx = u$.

Now by R-weakly commutativity of the pairs {A, S} and {B, T} we get

$m(Au, Su, a) = m(ASz, SAz, a) \leq m(Az, Sz, a) = 1$
$m(Bu, Su, a)=m(BTx, TBx, a) \leq m(Bx, Tx, a) = 1$

which gives $Au = Su$ and $Bu = Tu$.

Hence u is a common coincidence point of {A, S} and {B T} and thus the theorem proved.

REFERENCES

[1] A. Aghajani, M. Abbas, J. Roshan, Common fixed point of generalized weak contractive mappings in partiallyordered b-metric spaces, Math. Slovaca, 64 (2014), 941–960. https://doi.org/10.2478/s12175-014-0250-6.

[2] S. Czerwik, Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostrav. 1 (1993), 5–11.http://dml.cz/dmlcz/120469.

[3] S. Czerwik, Nonlinear set-valued contraction mappings in b-metric spaces, Atti Sem. Mat. Fis. Univ. Modena,46 (1998), 263–276. https://cir.nii.ac.jp/crid/1571980075066433280.

[4] S. Gahler, 2-metrische Raume und ihre topologische Struktur, Math. Nachr. 26 (1963), 115–148. https://doi.org/10.1002/mana.19630260109.

[5] M. Geraghty, On contractive mappings, Proc. Amer. Math. Soc. 40 (1993), 604–608.

[6] T.L. Hicks, B.E. Rhoades, A Banach type fixed point theorem, Math. Japon. 24 (1979), 327–330. https://cir.nii.ac.jp/crid/1572824499579575040.

[7] A.E. Bashirov, E.M. Kurpınar, A. Ozyapıcı, Multiplicative calculus and its applications, J. Math. Anal. Appl. 337(2008), 36-48.

[8] Z. Mustafa, V. Parvaneh, J.R. Roshan, et al. b2-Metric spaces and some fixed point theorems, Fixed PointTheory Appl. 2014 (2014), 144. https://doi.org/10.1186/1687-1812-2014-144.

[9] P. Singh, V. Singh, S. Singh, Some fixed points results using (ψ,φ)-generalized weakly contractive map in ageneralized 2-metric space, Adv. Fixed Point Theory, 13 (2023), 21. https://doi.org/10.28919/afpt/8218.

[10] S.H. Khan, P. Singh, S. Singh, et al. Fixed point results in generalized bi-2-metric spaces using θ-typecontractions, Contemp. Math. 5 (2024), 1257–1272

[11] M. Grossman, R. Katz, Non-Newtonian Calculus, Lee Press, Pigeon Cove, MA (1972).

[12] O. Ege and I. Karaca, Common fixed point results on complex valued Gb-metric spaces, Thai J. Math. 16, 3, 775-787, 2018.

[13] O. Ege, C. Park and A. H. Ansari, A different approach to complex valued Gb-metric spaces, Adv. Difference Equ. 2020, 152, 1-13, 2020.

[14] H. Huang, L. Paunovic, S. Radenovic, On some new fixed point results for rational Geragthy contractive mappings in ordered b-metric spaces, J Nonlinear Sci. Appl. 8(2015), No. 5, 800-807.

[15] B. Surender Reddy, S. Vijayabalaji, N. Thillaigovindan, K. Punniyamoorthy, Emerging Frameworks: 2-Multiplicative Metric and Normed Linear Spaces,Mathematics and Statistics 12(2): 199-203, 2024.

[16] Muhammad Usman Ali, Tayyab Kamran, Alia Kurdi;FIXED POINT THEOREMS IN B-MULTIPLICATIVE METRIC SPACES, U.P.B. Sci. Bull., Series A, Vol. 79, Iss. 3, 2017, 107-116.

[17] M. Iqbal, A. Batool, O. Ege and M. de la Sen, Fixed point of almost contraction in b-metric spaces, J. Math. 3218134, 1-6, 2020.

[18] M. Iqbal, A. Batool, O. Ege and M. de la Sen, Fixed point of generalized weak contraction in b-metric spaces, J. Funct. Spaces 2021, 2042162, 1-8, 2021.

[19] T. Kamran, M. Samreen and O.U Ain, A generalization of b-metric space and some fixed point theorems, Mathematics 5, 2, 2017.

[20] V. Ozturk, D. Turkoglu, Fixed points for generalized alpha-psi-contractions in b-metric spaces, J. Nonlinear Convex Anal. 16(2015), No. 10, 2059-2066.

[21] W. Shatanawi, A. Pitea, R. Lazovic, Contraction conditions using comparison functions on b-metric spaces, Fixed Point Theory Appl. 2014, Art. No. 135.

[22] M. Kumar, b2-Multiplicative Metric Space and A Fixed Point Theorem, Int J Stat. Appl. Math., Issue March 2025 Acceptted.

****