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RESEARCH ARTICLE

A ROBUST OBSERVER DESIGN FOR PASSIVITY-BASED SYNCHRONIZATION OF UNCERTAIN MODIFIED COLPITTS OSCILLATORS AND CIRCUIT SIMULATION

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ABSTRACT

This paper addresses passivity-based synchronization method of uncertain modified Colpitts oscillators. Considering the effect of external disturbances on system's parameters and nonlinear control inputs, a simple-workable robust passivity controller based on Lyapunov theory and linear matrix inequality (LMI) is designed for the output synchronization between a master system and a slave system consisting of uncertain modified Colpitts oscillators. A sufficient criterion is also investigated which can not only ensure the asymptotic stability of the solutions of the error dynamics but also reduce the influence of the disturbance within a prescribed level. The proposed controller can be obtained by solving a convex optimization problem presented by the LMI. In order to verify the effectiveness of the proposed control scheme, some numerical simulations are performed and a Pspice analog circuit implementation of the complete master-slave-controller system is presented.

Key words: Chaos, Colpitts oscillator, passivity, H_∞ synchronization, Lyapunov theory, linear matrix inequality.

INTRODUCTION

The synchronization of chaotic systems has been a very active field in nonlinear dynamics since its discovery [Fujisaka and Yamada, 1983; Pikovsky, 1984]. It has been observed in lasers, electronic circuits, chemical reactions, plasma discharges, fluids, natural systems [Schuster *et al.*, 1986; Wang *et al.,* 2000; Boccaletti *et al*., 2002]. In the recent years, the synchronization of spatially extended chaotic systems has attracted particular interest [Junge and Parlitz, 2000]. Many effective control methods have been proposed to achieve chaos synchronization, such as linear and nonlinear feedback controls [Yassen, 2005; Bai and Lonngren, 2000; Li *et al.*, 2007]. However, a certain number of drawbacks have been revealed in the practical implementation of most chaos-based secure communications algorithms. In particular, one of the basic issues of interest is the effect of parameter variation uncertainty and exoteric perturbation. Moreover, some or all of the system parameters are variable from time to time. Therefore, investigation of external disturbance perturbations in synchronization between drive and response chaotic systems has become an interesting and important research topic in nonlinear science. To overcome these difficulties, various adaptive synchronization schemes have been proposed and investigated [Liao and Lin, 1999; Zhang, 2004; Xiaojue, 2008; Ahn, 2009; Roopaei and Zolghadri, 2008; Park, 2006].

More recently, passivity-based control have been proved to be useful for the problem of motion coordination of multi-agent systems [Ren, 2007; Igarashi *et al*., 2007]. In Refs [Chopra and Spong, 2006] passivity-based control laws are presented for output synchronization of networks of passive nonlinear systems. Output synchronization is proved by employing the sum of storage functions as a Lyapunov function candidate. As shown in these references, passivity-based control enables one to handle communication delays and switching topology within a unified (energy-based) framework. Likewise, in [Arcak, 2007; Ihle *et al*., 2007] passivity is used to study the problem of steering the differences between the outputs of agents to a prescribed compact set, and to address the formation control or path synchronization problem. Passivity-based controls for chaotic Lu system and chaotic oscillations in power systems was proposed in [Kemih *et al.*, 2006] and [Wei and Luo, 2007] respectively. Kemih *et al*., proposed a Passivity-Based Synchronization of unified chaotic system in the work [Kemih *et al*, 2007]. Wang and Liu also applied this technique to design a controller to control a unified chaotic system to zero and desired equilibrium in [Wang and Liu, 2007]. Passitivity-based controls for hyperchaotic Lorenz system, hyperchaotic Chen system and nuclear spin generator chaotic system were proposed in [Wang and Liu, 2006; Kemih, 2009] respectively. Recently, Ahn, Jung and Joo [Ahn *et al.,* 2010] proposed a passivity-based synchronization between two different chaotic systems.

In 2000, Ababei *et al*. [Ababei and Marculescu, 2000] proposed a new topology of the Colpitts oscillator called "modified Colpitts oscillator" (further called MCO in this work) which was used in the qualitative numerical transmission of information. The particular feature of this oscillator is the real possibility to control chaos using a single resistor, without varying any parameter of the intrinsic Colpitts oscillator, which offers the possibility of an electronic analog or digital control on the system dynamics. We also note that there exists an important difference between existing

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MCO topologies and the classical Colpitts oscillator, which confers each one specific synchronization properties. In the case of the classical Colpitts oscillator, several studies have been achieved on the dynamical behavior and some control and synchronization strategies have been proposed [Fotsin and Daafouz, 2005; Guo-Hui, 2005]. More recently, synchronization of two identical modified Colpitts oscillators with structural perturbations have been proposed by Kammogne and Fotsin [Kammogne and Fotsin, 2011]. However, to the best of our knowledge, for the passivity-based synchronization between uncertain modified Colpitts oscillators and its implementation, there is no result in the literature so far, which still remains challenging. In this paper, considering the effect of exoteric perturbation on system's parameters, when only the system outputs are available, a passivity-based synchronization method is used in order to achieve robust output passivity-based synchronization for a MCO. Based on Lyapunov stability theory and linear matrix inequality (LMI), we get a new sufficient condition that can guarantee the robustness of the controller against the effect of exoteric perturbations and also a criterion of asymptotical stability of the solution of the error dynamics.

The LMI can be solved efficiently by using convex optimization algorithms [Boyd *et al*., 1994]. Throughout the paper, $\|x\|$ represents the Euclidian norm of the vector *x*, while $\|A\| = \sqrt{\rho(A^T A)}$ (where $\rho(Q)$ is the spectral radius of matrix Q) denotes the spectral norm of the matrix A. In the scientific literature on chaos control and synchronization, numerical examples are abundantly provided to verify synchronization algorithms meanwhile circuit implementations of the strategies are scarcely presented. For the sake of simplicity, it is always possible to begin with an analog simulation of the synchronization process which will confirm the practical feasibility of the scheme. The new MCO topology with operational amplifiers provided in this work spreads its scope of application in the way where the state variables are easily accessible in practice, which is an important feature of chaotic oscillators in telecommunications. Therefore, in this work, Pspice simulations will be carried on electronic analog models of our synchronizations strategies of the modified Colpitts oscillators. The rest of the work is organized as follows: In section 2 the model and its chaotic behaviour are presented. In section 3, we give some basic definitions and concepts of passivity. Section 4 introduces the H_∞ synchronization problem, while in section 5 we present our main result that consists of a new controller that is robust enough against the effect of exoteric perturbation of MCO. Numerical simulations are given in section 6 and circuit implementations are presented to show real applications of the method. Finally, conclusions are presented in section 7.

The modified Colpitts oscillator

The model

The simplest configuration of the MCO is shown on Fig.1 (a) [Ababei and Marculescu, 2000]. This circuit uses a bipolar junction transistor (BJT) as the gain element and a resonant network consisting of an inductor (L) and a pair of capacitors (C_1 and C_2). The resistor $\mathrm{R_d}$ is the additive element compared to the classical system. The nonlinear element is an NPN type BJT for which the simplified model shown on Fig.1 (b) consists of a non-linear voltage-controlled resistance R_E and a linear current-controlled current source I_E [Maggio and De Feo, 1999]. Parasite capacitances of the BJT can be neglected in the frequency range of oscillation since their effect would only be a frequency shift [Guo-Hui, 2005]. The V-I characteristic of the nonlinear resistor R_E (which corresponds normally to the emitter-base diode) and its approximate expression are defined as usual by:

$$I_E = I_S\left[\exp\left(\frac{eV_{BE}}{k_B T}\right) - 1\right] \approx I_S\left[\exp\left(\frac{eV_{BE}}{k_B T}\right)\right] \quad (1)$$

where I_S is the inverse saturation current of the emitter-base diode, *e* the elementary charge, k_B the Boltzmann constant and T the absolute temperature. The thermal voltage is given by: $V_T = \frac{K_B T}{e} \approx 26mV$. Here we set $I_S = 10^{-11} A$. The base-emitter voltage drop V_{BE} is given by:

$$V_{BE} = -u_2 - \mathrm{R_d}(i_L + i_B) \quad (2)$$

where i_B is the base current. Moreover, the relations between the currents are

$$i_L = i_1 + i_c, \; i_L = i_2 + i_3 \quad (3)$$

We assume $\alpha_F = 1$ where α_F is the common-base forward short-circuit current gain. This corresponds to neglecting the base current. Since we have $i_L >> i_B$, the simplified state equations for the schematic in Fig.1 (a) are the following [Kammogne and Fotsin, 2011].

$$\begin{aligned} \frac{du_1}{dt} &= \frac{i_L}{c_1} - \frac{I_E}{c_1} \\ \frac{du_2}{dt} &= \frac{i_L}{c_2} - \frac{u_2}{R_e c_2} - \frac{V_{CC}}{R_e c_2} \\ \frac{di_L}{dt} &= \frac{V_{CC}}{L} - \frac{(R_C + \mathrm{R_d})}{L} i_L - \frac{u_1}{L} - \frac{u_2}{L} \end{aligned} \quad (4)$$

We now introduce a set of dimensionless state variables (*x, y, z*) where we normalize voltages, currents and time according to the following relations:

$$u_1 = V_{CC}x, \; \theta = \sqrt{Lc_1}, \; t = \theta\tau, \; u_2 = V_{CC}y, \; i_L = \frac{V_{CC}}{\sqrt{L/c_1}} z \quad (5)$$

Using (2) ,(3) and (5), the system of Eqs. (4) can be rewritten in the following form :

$$\frac{dx}{d\tau} = z - a_2 \exp(-az - by)$$
$$\frac{dy}{d\tau} = -b_0 - b_0 y + z \quad (6)$$
$$\frac{dz}{d\tau} = 1 - x - y - c_{11} z$$

With $b_0 = \frac{\theta}{R_e C_2}, b = \frac{\theta I_S}{C_2 V_{CC}}, c_{11} = \frac{\theta(R_C + R_d)}{L}, a = \frac{V_{CC}}{V_T}\frac{R_d}{\sqrt{L/C_1}}, a_2 = \frac{I_S \theta}{C_1 V_{CC}}$

Let's select the parameters: L=98.5μH, $R_C = 35\Omega$, $R_E = 400\Omega$, $V_{CC} = V_{EE} = 5V$, $C_1 = C_2 = 54nF$ $R_d = 0.5\Omega$ which yields $a = 2.251362,\ b = 192.3,\ b_0 = 0.1064814815$ $c_{11} = 0.934,\ a_2 = 8.518518.10^{-11}$, the system shows a chaotic behavior characterized by a maximal Lyapunov exponent $\lambda_{\max} = 0.06393$ which confirms occurrence of chaotic oscillations.

Passivity

Consider the following differential equation

$$\dot{X}(t) = \left(F(X(t)) + \Delta_F(X(t))\right) + \left(G(X(t)) + \Delta_G(X(t))U(t)\right) \quad (7)$$
$$Y(t) = H(X(t))$$

where $X(t) \in R^n$ is the state variable, $U(t) \in R^m$ is the external input, $Y(t) \in \mathbb{R}^m$ is the output, F and G are smooth vector fields and H is a smooth mapping. $\Delta_F(X(t))$ and $\Delta_G(X(t))$ denotes parametric uncertainties. Without loss of generality, we suppose that the vector field F has at least one equilibrium point. The notion of passivity can be described as follows:

Definition 1

If there exist two nonnegative constants β and γ, a positive semi-definite function $S(X(t))$ and a nonnegative function $R(t)$ such that

$$\int_0^t U^T(\tau)Y(\tau)d\tau + \beta \geq \int_0^t S(X(\tau))d\tau - \gamma^2 \int_0^t R(\tau)d\tau \quad \forall t \geq 0 \quad (8)$$

The system (7) is said to be passive in the presence of disturbance from the external input $U(t)$ to the output $Y(t)$. The Physical meaning of a passive system is that the energy of the nonlinear system (7) can be increased only through the supply from the external source. In other words, a passive system cannot store more energy than it is supplied. A Passive system is naturally a stable system. Passive systems exploit the input-output relationship based on energy-related considerations to analyze stability properties. The following statement describes a basic stabilizability property of passive systems.

Corollary1: [Byrnes *et al.*, 1991)]

If we neglect the parametric variation of the system ($\Delta_F(X(t)) \approx 0$ and $\Delta_G(X(t)) \approx 0$) then, the relation (8) leads to

$$\int_0^t U^T(\tau)Y(\tau)d\tau + \beta \geq \int_0^t S(X(\tau))d\tau, \quad \forall t \geq 0 \quad (9)$$

Then, the systems (7) - (8) is called *strictly passive* if $S(X) > 0$ and *lossless* if $S(X) = 0$

Definition 2

Suppose the system (7) is passive. Let $\phi(\cdot)$ be any smooth function such that $\phi(0) = 0$ satisfying the following condition:

$$Y^T(t)\phi(Y(t)) - R(t) > 0 \quad (10)$$

For each nonzero $Y(t)$, the control law $U(t) = -\phi(Y(t))$ asymptotically stabilizes the equilibrium point of the system (7).

System formulation

Let a class of MCO be defined as follows

$$\dot{X}(\tau) = AX(\tau) + C\varphi(X(\tau)) + B \quad (11)$$
$$\tilde{y}(\tau) = DX(\tau)$$

$X \in R^3$ is the system state, $\tilde{y} \in R^3$ is the output vector, $A, C, D \in \mathbb{R}^{3\times3}$ and $B \in R^{3\times1}$ are known matrices. $\varphi: R^3 \to R^3$ is a nonlinear continuously differentiable function. Considering the effect of the exoteric perturbation on system's parameters, the master's system is given by

$$\dot{X}(\tau) = (A + \Delta A_1(\tau))X(\tau) + C\varphi(X(\tau)) + B + \Delta B_1(\tau) \quad (12)$$
$$\tilde{y}_m(\tau) = DX(\tau)$$

And the slave system is given by

$$\dot{Y}(\tau) = (A + \Delta A_2(\tau))Y(\tau) + C\varphi(Y(\tau)) + B + \Delta B_2(\tau) + u(\tau) \quad (13)$$
$$\tilde{y}_s(\tau) = DY(\tau)$$

where $\Delta A_1(\tau)$ and $\Delta A_2(\tau)$ are bounded structural variations of the system which satisfy the condition $\|\Delta A_i(\tau)\| \leq \delta$, $i = 1, 2$ (δ being a positive constant), $\Delta B_1(\tau)$ and $\Delta B_2(\tau)$ are perturbed matrix such that $\|\Delta A_i(\tau)\| \geq \|\Delta B_i(\tau)\|$; $i = 1, 2$ and $u(\tau)$ is the nonlinear controller. $\tilde{y}_m$ and $\tilde{y}_s$ are respectively the output vectors of the master and the slave systems. Our goal is to design a simple appropriate controller $u(\tau)$ such that the trajectory of the slave system (13) asymptotically approaches the master system (12) and finally achieves synchronization. Define the synchronization error vector as

$$e(\tau) = Y(\tau) - X(\tau), \quad y_e(\tau) = \tilde{y}_s(\tau) - \tilde{y}_m(\tau) \tag{14}$$

Hence the error dynamical system is

$$\begin{aligned} &\dot{e}(\tau) = Ae(\tau) + \Delta A_2(\tau)Y(\tau) - \Delta A_1(\tau)X(\tau) + C\left(\varphi(Y(\tau) - \varphi(X(\tau))\right) + \Delta B_2(\tau) - \Delta B_1(\tau) + u(\tau) \\ &y_e(\tau) = De(\tau) \end{aligned} \tag{15}$$

Definition 3: *Systems (12) and (13) asymptotically synchronize if the synchronization error* $e(\tau)$ *satisfies* $\lim_{\tau\to\infty}\|e(\tau)\| = 0$

Assumption 1

We define $\omega(\tau) = \Delta B_2(\tau) - \Delta B_1(\tau)$ and $h(\tau) = \Delta A_2(\tau) - \Delta A_1(\tau)$. Let $\chi^T(\tau)\chi(\tau) = \omega^T(\tau)\omega(\tau) - h^T(\tau)h(\tau)$ and assume $\chi(\tau) \in L_2$, that is $\int_0^\infty \chi^T(\tau)\chi(\tau)d\tau \le \infty$.

The H_∞ performance problem will be considered, which can be formulated as follows: Design the controller $u(\tau)$ such that:

- If $\chi(\tau) = 0$

$$\int_0^\tau e^T(t)Qe(t)dt \le \int_0^\tau \hat{y}^T(t)\xi(t)dt + \beta \tag{16}$$

where Q is a positive symmetric matrix, $\xi(\tau)$ is an external input signal, $\hat{y}(\tau)$ is a positive function. For a particular selection of $\xi(\tau)$ [Ahn *et al.*, 2010], the error system (15) will be asymptotically stable; Given a positive scalar γ and under zero initial conditions,

- If $\chi(\tau) \neq 0$ the following conditions will hold:

$$\int_0^\infty e^T(\tau)Qe(\tau)d\tau \le \gamma^2\int_0^\infty \chi^T(\tau)\chi(\tau)d\tau + \int_0^\infty \hat{y}^T(\tau)\xi(\tau)d\tau \tag{17}$$

γ is the disturbance attenuation level [Stoorvogel, 1992] which is defined by

$$\gamma = \frac{\sqrt{\int_0^\infty e^T(\tau)Qe(\tau)d\tau}}{\sqrt{\int_0^\infty \chi^T(\tau)\chi(\tau)d\tau}}. \tag{18}$$

One part of this work is to determine a suitable gain matrix (further called *K*) such that the performance index (18) is within the upper bound, that is,

$$\sup_{\substack{\chi(\tau)\neq 0, y(\tau)\neq 0 \\ \chi(\tau)\in L_2, \hat{y}(\tau)\in L_2}} \left(\frac{\|y_e(\tau)\|_2}{\|\chi(\tau)\|_2 + \|\hat{y}(\tau)\|_2}\right) \square \ \gamma \tag{19}$$

Lemma 1 (Schur Complements):

For a given symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \text{ where } S_{11} = S_{11}^T,\ S_{12} = S_{21}^T \text{ and } S_{22} = S_{22}^T$$

The condition $S < 0$ is equivalent to $S_{22} < 0$ and $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$

Main results

Assumption 2: We can choose the controller in this form

$$u(\tau) = -KDe(\tau) - C\left(\varphi(Y(\tau)) - \varphi(X(\tau))\right) + \xi(\tau) \tag{20}$$

where *K* is the gain matrix. One advantage of this type of controller is that it can be easily constructed through time varying resistors, capacitors or operational amplifier and their combinations, or using a digital signal processor together with the appropriate converters (an example of construction is presented at the end).

Assumption 3: *For a given* $\Omega > 0$ *and* $Q = Q^T > 0$, *if there exist* $F = F^T > 0$ *and suitable matrix* L *such that the following equality holds*

$$\begin{bmatrix} AF + FA^T - L^T - L - \eta^{-1}\delta^2 I & F & I \\ F & -\Omega^{-2}I & 0 \\ I & 0 & -Q^{-1} \end{bmatrix} < 0 \tag{21}$$

then the passivity-based-synchronization is achieved under the controller (20).

Theorem 1. *Systems (12) and (13) approach synchronization under the condition (20)*

Proof. The closed loop error system with the control (19) can be written as

$$\dot{e}(\tau) = (A - KD)e(\tau) + \Delta A_2(\tau)Y(\tau) - \Delta A_1(\tau)X(\tau) + \Delta B_2(\tau) - \Delta B_1(\tau) + \xi(\tau) \tag{22}$$

Consider the Lyapunov function

$$V(\tau) = e^T(\tau)Pe(\tau). \tag{23}$$

Where $P = P^T > 0$. It obvious that V is a positive function for all $\tau \ge 0$.

Differentiating *V* with respect to time τ one obtains

$$\begin{aligned} \dot{V}(\tau) &= \dot{e}^T(\tau)Pe(\tau) + e^T(\tau)P\dot{e}(\tau) \\ &= \left[(A-KD)e(\tau) + \Delta A_2(\tau)Y(\tau) - \Delta A_1(\tau)X(\tau) + \Delta B_2(\tau) - \Delta B_1(\tau)\right]^T Pe(\tau) + e^T(\tau)P\left[(A-KD)e(\tau) + \right. \\ &\quad \left. + \Delta A_2(\tau)Y(\tau) - \Delta A_1(\tau)X(\tau) + \Delta B_2(\tau) - \Delta B_1(\tau)\right]^T + 2e^T(\tau)P\xi(\tau) \\ &= e^T\left((A-KD)^T P + P(A-KD)\right)e(\tau) + 2e^T(\tau)P\left(\Delta A_2(\tau)Y(\tau) - \Delta A_1(\tau)X(\tau)\right) + e^T(\tau)P \\ &\quad + \left(\Delta B_2(\tau) - \Delta B_1(\tau)\right) + \left(\Delta B_2(\tau) - \Delta B_1(\tau)\right)^T Pe(\tau) + 2e^T(\tau)P\xi(\tau) \\ &= e^T\left((A-KD)^T P + P(A-KD)\right)e(\tau) + 2e^T(\tau)P\Delta A_2 e(\tau) + 2e^T(\tau)P\left(\Delta A_2(\tau) - \Delta A_1(\tau)\right)X(\tau) + \\ &\quad + e^T(\tau)P\left(\Delta B_2(\tau) - \Delta B_1(\tau)\right) + \left(\Delta B_2(\tau) - \Delta B_1(\tau)\right)^T Pe(\tau) + 2e^T(\tau)P\xi(\tau) \end{aligned} \tag{24}$$

Proposition

For any matrices $Q_1 \in R^{n\times m}$, $Q_2 \in R^{n\times m}$, $\Theta = \Theta^T > 0$, and $\Theta \in R^{n\times m}$ we have the following inequalities.

$$Q_1^T Q_2 + Q_2^T Q_1 \le Q_1^T \Theta Q_1 + Q_2^T \Theta^{-1} Q_2 \quad (25)$$

This proposition holds:

$$e^T P\left(\Delta B_2(\tau) - \Delta B_1(\tau)\right) + \left(\Delta B_2(\tau) - \Delta B_1(\tau)\right)^T Pe \le \gamma^{-2} e(\tau)^T P^2 e(t) + \gamma^2 \omega(\tau)^T \omega(\tau) \quad (26)$$

$$2e^T(\tau) P \Delta A_2 e(\tau) \le \eta d^2 e^T(\tau) P^2 e(\tau) + \eta^{-1}\delta^2 e^T(\tau) e(\tau)$$

For any time $\tau \ge 0$, the system state is bounded (as the system is chaotic), hence we may write $\|X(\tau)\| \le d$. Here d, η and γ are positives constants. The Relation (24) leads to

$$\begin{aligned}\dot V &\le e^T(\tau)\left((A-KD)^T P + P(A-KD)\right)e(\tau) + \gamma^{-2} e(\tau)^T P^2 e(t) + \gamma^2 \omega(\tau)^T \omega(\tau) + \eta d^2 e^T(\tau) P^2 e(\tau) + \\ &\quad + \eta^{-1}\delta^2 e^T(\tau) e(\tau) + \gamma^{-2} d^2 e^T(\tau) P^2 e(\tau) + \gamma^2 h^T(\tau) h(\tau) + 2e^T(\tau) P \xi(\tau) \\ &\le e^T(\tau)\left((A-KD)^T P + P(A-KD)\right)e(\tau) + \gamma^{-2}P^2 + \eta d^2 P^2 + \eta^{-1}\delta^2 I + \gamma^{-2} d^2 P^2 + \gamma^2 \omega(\tau)^T \omega(\tau) + \\ &\quad + \gamma^2 h^T(\tau) h(\tau) + 2e^T(\tau) P \xi(\tau) \\ &\le e^T(\tau)\left((A-KD)^T P + P(A-KD) + (\gamma^{-2} + \gamma^{-2} d^2 + \eta d^2) P^2 + \eta^{-1}\delta^2 I\right)e(\tau) + \\ &\quad + \gamma^2\left(\omega(\tau)^T \omega(\tau) + h^T(\tau) h(\tau)\right) + 2e^T(\tau) P \xi(\tau)\end{aligned}$$

Now let $\Omega^2 = \gamma^{-2} + \gamma^{-2} d^2 + \eta d^2$, $\hat y^T(\tau) = 2e^T(\tau) P$ and $\chi(\tau)^T \chi(\tau) = \omega(\tau)^T \omega(\tau) + h^T h$

If the matrix inequality

$$(A-KD)^T P + P(A-KD) + \Omega^2 P^2 + \eta^{-1}\delta^2 I + Q < 0 \quad (27)$$

is satisfied, we have

$$\dot V \le -e^T Q e + \gamma^2 \chi(\tau)^T \chi(\tau) + \hat y^T(\tau)\xi(\tau) \quad (28)$$

It is easy to find that if $\chi(\tau) = 0$ then

$$\dot V(\tau) < -\lambda_{\min}(Q)\|e(\tau)\|^2 + \hat y^T(\tau)\xi(\tau) \qquad (e(\tau) \ne 0) \quad (29)$$

Integrating both sides of (29) from 0 to τ gives

$$V(e(\tau)) - V(e(0)) < -\int_0^\tau e^T(t) Q e(t) dt + \int_0^\tau \hat y^T(t)\xi(t) dt$$

Let $\beta = V(e(0))$ since $\beta = V(e(0)) > 0$,

$$\int_0^\tau \hat y^T(t)\xi(t) dt + \beta > \int_0^\tau e^T(t) Q e(t) dt + V(e(t)) \ge \int_0^\tau e^T(t) Q e(t) dt \quad (30)$$

The relation (30) satisfies the passivity definition (9)
If $\chi(\tau) \ne 0$, integrating both sides of Ineq.(28) gives

$$V(e(\tau)) - V(e(0)) < -\int_0^\tau e(t)^T Q e(t) dt + \gamma^2 \int_0^\infty \chi(t)^T \chi(t) dt + \int_0^\tau \hat y^T(t)\xi(t) dt \quad (31)$$

Let
$$R(t) = \chi(t)^T \chi(t) \quad (32)$$

we obtain

$$\int_0^\tau \hat y^T(t)\xi(t) dt + \beta > \int_0^\tau e(t)^T Q e(t) dt - \gamma^2 \int_0^\infty R(t) dt + V(e(\tau)) \ge \int_0^\tau e(t)^T Q e(t) dt - \gamma^2 \int_0^\tau R(t) dt \quad (33)$$

Since $V(e(\tau)) \ge 0$ we have the relation in Ineq. (8). This completes the proof. For solving inequality. (27), let's consider the lemma1 which can be easily solved by using the MATLAB LMI Control Toolbox.

Theorem 2: Given three positive constants η, δ, γ and take into account the lemma 1, then the matrix inequality (27) is equivalent to

$$\begin{bmatrix} (A-KD)^T P + P(A-KD) + \eta^{-1}\delta^2 I & P & I \\ P & -\Omega^{-2} I & 0 \\ I & 0 & -Q^{-1} \end{bmatrix} < 0 \quad (34)$$

Pre- and post-multiplying Ineq. (34) by the $\mathrm{diag}(P^{-1}, I, I)$ and introducing a change of variables such as $F = P^{-1}$ and $L = KDP^{-1}$, Ineq. (32) is equivalently changed into the LMI form (21). Then the gain matrix of the control input $u(\tau)$ is given by $K = LF^{-1}D^{-1}$. This completes the proof. According to the definition 2, once the error system (17) has been rendered passive, the external input signal $\xi(\tau) = -\phi(\hat y(\tau))$ satisfying $\phi(0) = 0$ and $\hat y^T(\tau)\phi(\hat y(\tau)) > 0$ for each nonzero $\hat y(\tau)$ approaches asymptotically stabilizes the error (17).

Theorem 3: If the external input signal $\xi(\tau)$ is selected as

$$\xi(\tau) = -\gamma^2 \hat y(\tau) \quad (35)$$

The closed-loop error system approaches asymptotically the synchronization.

Proof: For $\xi(\tau) = -\gamma^2 \hat y(\tau)$, the time derivative of $\dot V(e(t))$ satisfies

$$\begin{aligned}\dot V(e(\tau)) &\le -e^T Q e + \gamma^2 \chi(\tau)^T \chi(\tau) - \gamma^2 \hat y^T(\tau)\hat y(\tau) \\ &\le -e^T Q e - \gamma^2\left(\hat y^T(\tau)\hat y(\tau) - \chi(\tau)^T \chi(\tau)\right)\end{aligned}$$

Considering relation (10) of the definition 2, one obtains

$$\dot V(e(\tau)) \le 0 \quad (36)$$

This guarantees the asymptotical stability from Lyapunov stability theory. This completes the proof.

Simulation investigation

Numerical analysis

In this section, numerical results are given to verify the effectiveness of the proposed method. The master and slave system are expressed as on equations (12) and (13), with

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -b_0 & 1 \\ -1 & -1 & -c_{11} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -b_0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \varphi(X) = \begin{bmatrix} a_2 \exp(-az - by) \\ 0 \\ 0 \end{bmatrix},$$

$$D = \sigma \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where $\sigma \neq 0$. For all simulations we take $R_d = 0.5\Omega$ which corresponds to a chaotic state of the MCO system. The output is defined as $\hat{y}(\tau) \cong 2Pe(\tau)$. Fig.2 shows the time history of synchronization of the MCO of system (6) without disturbances. We select structural variations simply as follows:

$$\Delta A_1 = \phi(\tau)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 8 \end{bmatrix},\ \Delta A_2 = \varphi(\tau)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad (37)$$

where $\phi(\tau) \in [-1, 1]$ and $\varphi(\tau) \in [-1, 1]$

Let us select the perturbed matrices simply in the following form

$$\Delta B_1(\tau) = \begin{bmatrix} 0 \\ \varepsilon_1(\tau) \\ 0 \end{bmatrix} \text{ and } \Delta B_2(\tau) = \begin{bmatrix} \varepsilon_2(\tau) \\ 0 \\ 0 \end{bmatrix} \quad (38)$$

Firstly, some constants are given in the following: $\gamma = 0.75,\ d = 2,\ \eta = 0.18,\ \delta = 0.15, \sigma = 1$.

$$\phi(\tau)=\sin(12\tau),\ \varphi(\tau)=\sin(23\tau) \text{ and } \varepsilon_2(\tau)=0.2\sin(6\tau)+0.8\sin(8\tau)+0.3\sin(12\tau).$$

Simultaneously, we adopt $\varepsilon_1(\tau)$ as a white Gaussian noise in the range [-4, 4] whose histogram is provided in Fig.3. The initial conditions are $(x_0, y_0, z_0) = (17\times10^{-4}, 12.10^{-4}, 13.10^{-4})$ for the master system and $(2.10^{-7}, 2.10^{-2}, 2.10^{-3})$ for the slave system. Fig. 4(a) and 4(b) respectively present the output errors $y_{e_x}(\tau)$, $y_{e_y}(\tau)$ and $y_{e_z}(\tau)$ with disturbance when any control input is not applied. In order to verify the convergence of the trajectory, let us introduce the quadratic gap of the error system: $y_{e_q}(\tau) = \|De(\tau)\|$. Fig. 4(c) shows the history of $y_{e_q}(\tau)$. It is found that, when the system parameters are perturbed, the errors with any control input cannot converge to zero as time τ tends to infinity. In order to realise the output synchronization, let's consider our approach: By use of the MATLAB LMI toolbox, from (21) with $Q = 15 \times I$, where $I \in R^{3\times3}$ is an identity matrix, we derive the following matrices

$$P = 10^{-3}\begin{bmatrix} 0.1090 & 0.0000 & 0.0000 \\ 0.000 & 0.1090 & 0.0000 \\ 0.0000 & 0.0000 & 0.1090 \end{bmatrix} \text{ and } K = \begin{bmatrix} -0.4843 & 0.0000 & 0.0000 \\ 0.0000 & 0.3778 & 0.0000 \\ 0.0000 & 0.0000 & -0.4497 \end{bmatrix} \quad (39)$$

Fig. 4(a), Fig. 5(b) and Fig. 5(c) show the output synchronization errors $y_{e_x}(\tau), y_{e_y}(\tau), y_{e_z}(\tau)$ and the error norm $y_{e_q}(\tau)$ when the presented method is applied. A simple observation shows that the present controls reduce efficiently the amplitude of synchronization errors. The maximal value of $y_{e_q}(\tau)$ after the transient period is 0.0016, which is very small compared with $\gamma = 0.15$. At this time, we can conclude that robust positivity output passivity-based-synchronization is achieved via the proposed method. Fig.6 shows the time history of synchronization of the MCO of system (6) without disturbances obtained with the gain of matrix (39) and the following initial conditions $(x_0, y_0, z_0) = (17\times10^{-4}, 14\times10^{-4}, 34\times10^{-4})$ for the master system and $(17\times10^{-4}, 14\times10^{-4}, 34\times10^{-4})$ for the slave system. It clearly appears that after a transient oscillatory period, the synchronization errors converge to zero. Fig. (6a), (6b) and (6c) show that the synchronization is achieved at $\tau = 27$. Secondly, we select constants $\gamma = 0.75,\ d = 2,\ \eta = 0.18,\ \delta = 0.15, \sigma = 0.2$

$$\phi(\tau)=\sin(12\tau),\ \varphi(\tau)=\sin(23\tau) \text{ and } \varepsilon_2(\tau)=0.2\sin(6\tau)+0.8\sin(8\tau)+0.3\sin(12\tau),$$

$\varepsilon_1(\tau)$ is taken as uniform distribution random noise in the range [-1,1] provide in Fig. 7. Fig. 8(a), Fig.8 (b), Fig.8(c) and Fig. 8(d) depicts the synchronization errors and the error norm when any control input is not applied with the same initial conditions adopted for both the master and slave system: $(10^{-7}, 2\times10^{5}, 3\times10^{4})$. We also observe that the errors with any control input cannot converge to zero with the passage of time τ. In order to observe H_∞ performance with attenuation, we derive with the aid of MATLAB LMI toolbox the following matrices:

$$P = \begin{bmatrix} 0.0960 & 0.0000 & 0.0000 \\ 0.0000 & 0.0960 & 0.0000 \\ 0.0000 & 0.0000 & 0.0960 \end{bmatrix}; K = \begin{bmatrix} 2.1867 & 1.4644 & -0.6225 \\ -1.4644 & 1.6543 & -0.2250 \\ 0.6225 & 0.2250 & -2.4833 \end{bmatrix} \quad (40)$$

Figures 8(a), 8(b), 8(c) and 8(d) respectively present the synchronization errors $y_{e_x}(\tau), y_{e_y}(\tau), y_{e_z}(\tau)$ and the error norm $y_{e_q}(\tau)$ based on the proposed method. It clearly appears that the errors system change approximately in the range $[-0.0015,\ 0.0015]$. The amplitude of error norm is 0.0035 which is to be compared with $\gamma = 0.75$. Hence, the fluctuations of the errors are very small as compared to the dynamical ranges of the system state variables. Thus, our control scheme in the presence of structural perturbations is robust enough, which is consistent with our theoretical analysis.

Circuit implementation

MCO with operational amplifiers

Although the MCO is a relatively simple oscillator, access to some state variables of the system can practically turn to a delicate task, because of the high frequency of operation. In order to simplify access and manipulation of the three state variables x, y and z, we propose in this section an electronic analog operational amplifiers-based model of the MCO (further called MCOOA). The circuit includes five operational amplifiers for integration and inversion operations. The device features are high slew rates, low input bias and offset currents and low offset voltage temperature coefficient. This is significant to reduce sensitivity to circuit parameter values. Hence we obtain a particularly elegant circuit in which the required nonlinearity is provided by a single silicon diode. This diode model is here taken as

$$f(y,z) = \frac{R}{R_D} I_0 \exp\left[\frac{0.026(-az-by)}{V_T} - 1\right] \approx \frac{R}{R_D} I_0 \exp\left[\frac{0.026(-az-by)}{V_T}\right] \quad (41)$$

with $I_0 = 10^{-12} A$ and $V_T = 0.026V$, giving a forward voltage drop of approximately $0.6V$ at room temperature, as it is well-known for normal silicon diodes. The components values and the voltage sources of this circuit are taken as Cx=Cy=Cz=10nF, (we recall that the voltages V_{Cx}, V_{Cy}, and V_{Cz} represents respectively the variables x, y and z). R=10KΩ, RD= 117.39Ω, R1=170.837KΩ, R2=2KΩ, R3=R4=93.913KΩ, R5=10.7KΩ, V1=V2=1V. The operational amplifiers are supplied with Vcc=+15V and Vss=-15V. The system parameters are given by

$$a_2 = \frac{R}{R_D} I_0, \qquad a = \frac{R}{0.026R_1}, \qquad b = \frac{R}{0.026R_2},$$
$$c_{11} = \frac{1}{10^4 R_5 C_3}, \qquad b_0 = \frac{1}{10^4 R_3 C_2} = \frac{1}{10^4 R_4 C_2}.$$

The resulting phase space plot of this circuit shows a good agreement with the Matlab simulations. The time history of the trajectories and phase portrait are provide in Fig. 11. It is noted that, as a new chaotic jerk circuit proposed by J.C. Sprott [Sprott, 2011], the components values were chosen to make the circuit oscillate in the audio range (see frequency spectrum of the x output in Fig.12), so that the chaos can be easily heard and displayed on any oscilloscope, although the frequency can be scaled up or down as desired over several decades. No attempt was made to find the upper frequency limit of operation since that would largely depend on the choice of components.

Pspice synchronization

In this part, we present a prototype of the passivity-based synchronization scheme of modified Colpitts oscillator in practical situations when no disturbance is taken into account. The components of the controller are given by

$$\begin{aligned} u_x(t) &= -\mu_1 e_1(t) - Ca_2\left[\exp\left(-az_s - by_s\right) - \exp\left(-az_m - by_m\right)\right] \\ u_y(t) &= -\mu_2 e_2(t) \\ u_z(t) &= -\mu_3 e_3(t) \end{aligned} \quad (42)$$

where

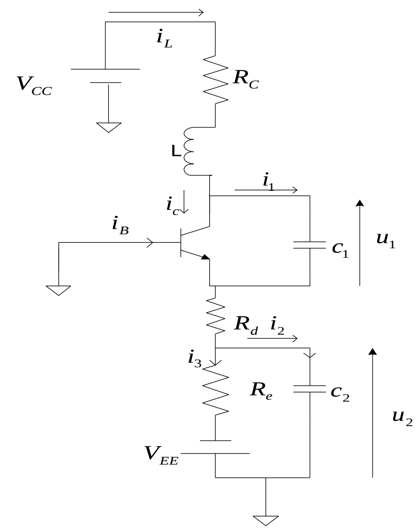
$\mu_1 = K_1\sigma + 2\gamma^2 P_1,\ \mu_2 = K_2\sigma + 2\gamma^2 P_2,\ \mu_3 = K_3\sigma + 2\gamma^2 P_3$

and $P_1 = P_2 = P_3 = 0.1090 \times 10^{-3}$

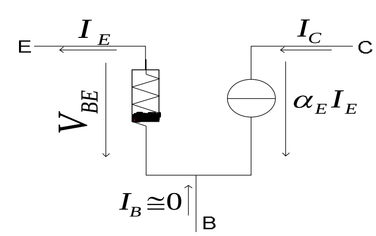
The component values of the controller are taken as R=10KΩ, R_z=22.3KΩ, R_y=26.5KΩ, R_{x2}=117391559 MΩ, R_{X1}=20.7KΩ. The parameters are given by

$$\mu_1 = \frac{1}{10^4 R_{X1} C_1};\ \mu_2 = \frac{1}{10^4 R_Y C_2};\ \mu_3 = \frac{1}{10^4 R_Z C_3};\ a_2 = \frac{1}{10^4 R_{X2} C_1}$$

Fig. 13 presents the Pspice realization of the passivity-based synchronization process, while Fig.14 shows the time variations of the synchronization errors $V_{CXm}(t) - V_{CXe}(t)$, $V_{CYm}(t) - V_{CYe}(t)$ and $V_{CZm}(t) - V_{CZe}(t)$ from a Pspice simulation (where subscript "*m*" indicate voltages from the master oscillator and "*e*" the voltages the from the slave system).



(a)



(b)

Fig. 1. Circuit model: (a) Schematic of the Colpitts oscillator. (b) BJT model in common base configuration

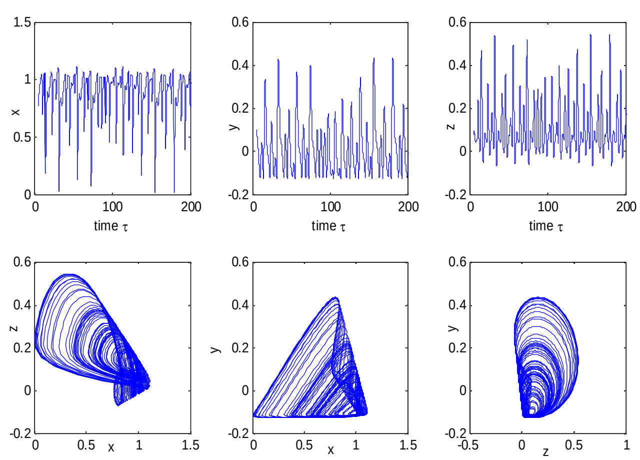


Fig. 2 (a) time history of MCO (b) Phase portrait (*x* versus *z*, *x* versus *y*, *z* versus *y*)

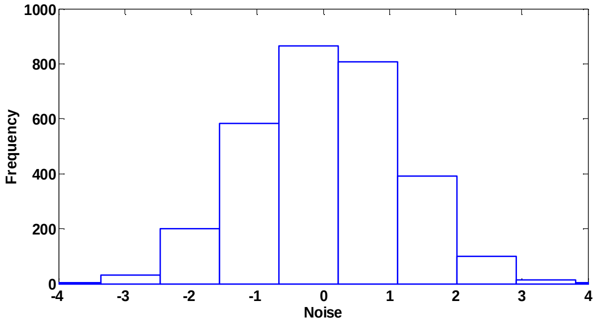
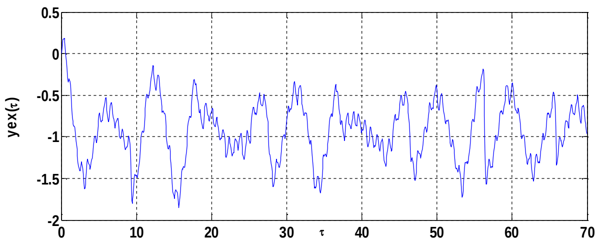
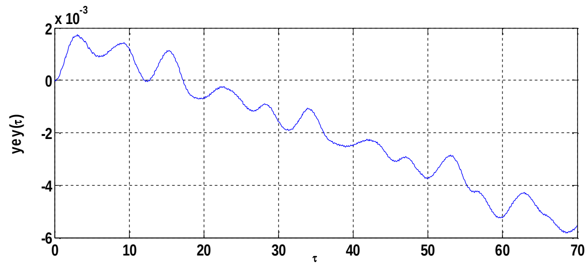


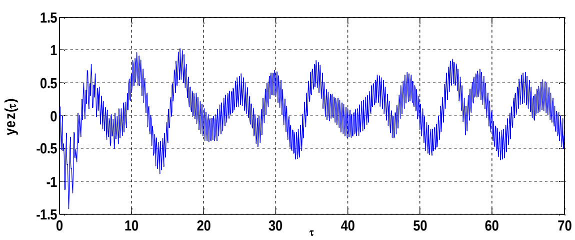
Fig. 3. Histogram of uniform random noise in the range [-4, 4]



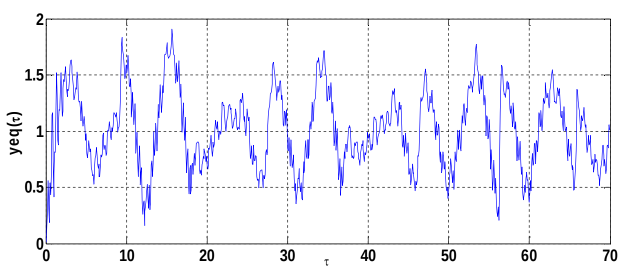
(a)



(b)

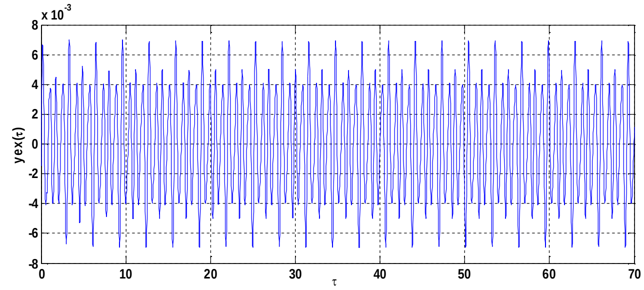


(c)

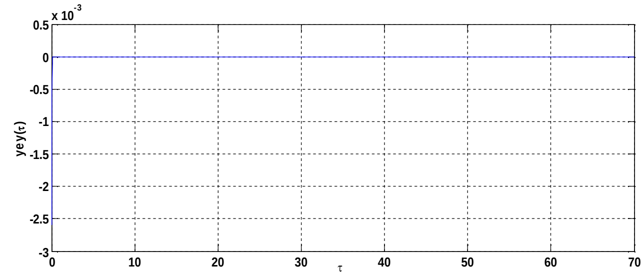


(d)

Fig. 4 Time evolution of the synchronization errors with disturbance when no control input is applied. (a): $y_{e_x}(\tau)$ **; (b):** $y_{e_z}(\tau)$ **; (c):** $y_{e_x}(\tau)$ **and (d) :** $y_{e_q}(\tau)$



(a)



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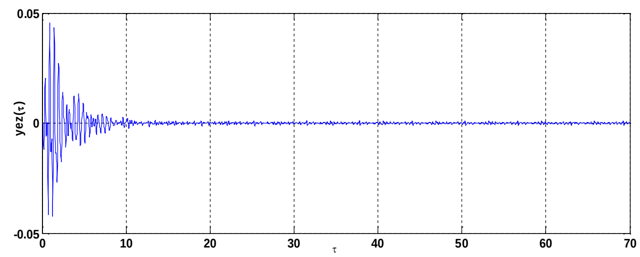
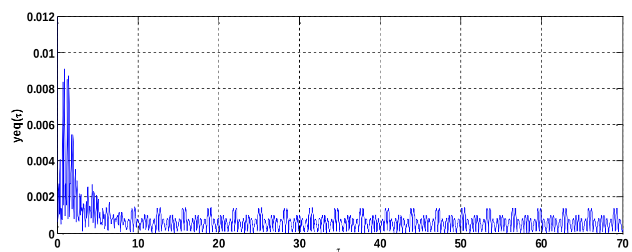


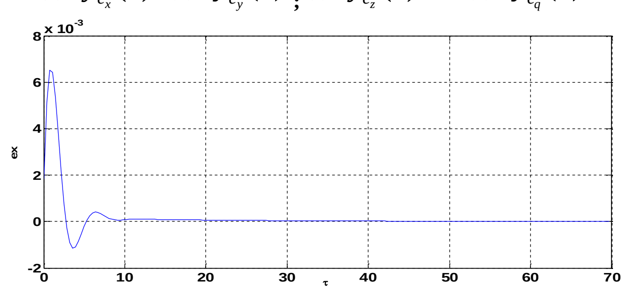
(c)



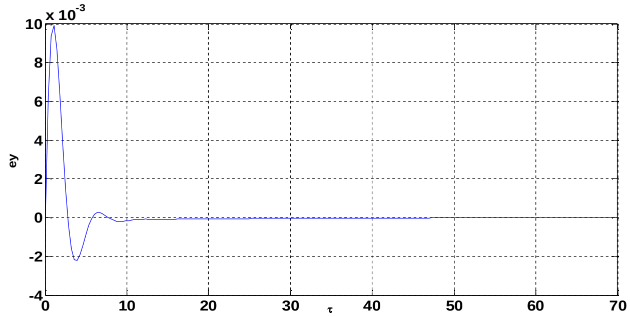
(d)

Fig. 5. Time evolution of the synchronization errors in presence of the disturbance.

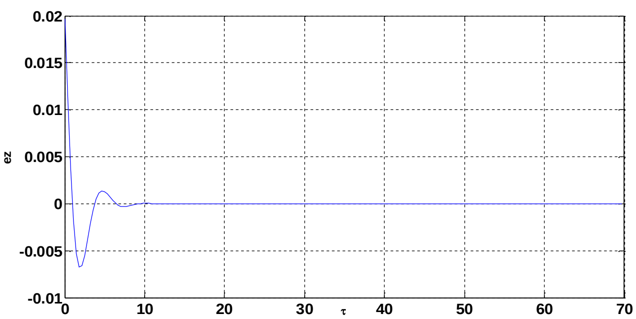
(a): $y_{e_x}(\tau)$ **; (b):** $y_{e_y}(\tau)$ **; (c):** $y_{e_z}(\tau)$ **and (d):** $y_{e_q}(\tau)$



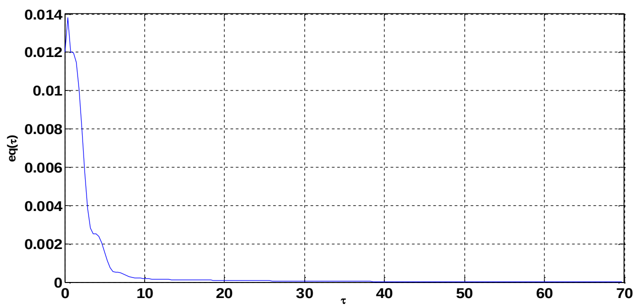
(a)



(b)



(c)



(d)

Fig. 6 Time evolution of the synchronization errors without disturbances. (a): $y_{e_x}(\tau)$ **;(b):** $y_{e_y}(\tau)$ **(c):** $y_{e_z}(\tau)$ **; (d) :** $y_{e_q}(\tau)$

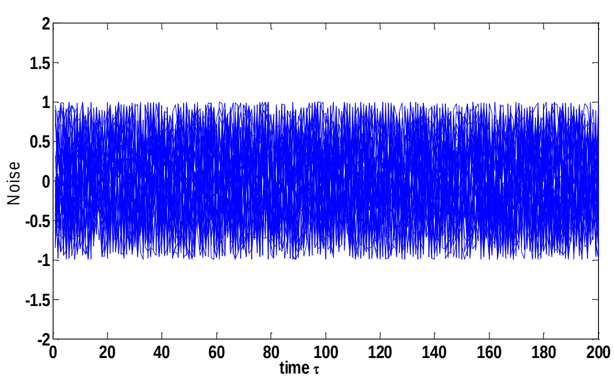
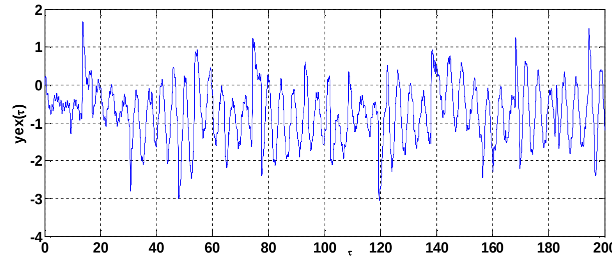
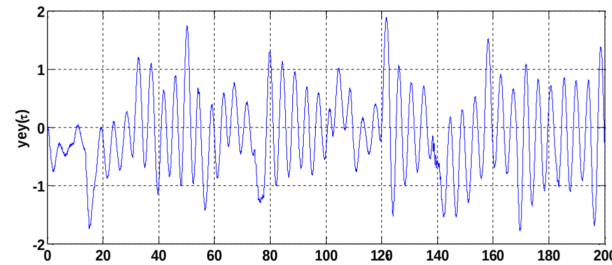


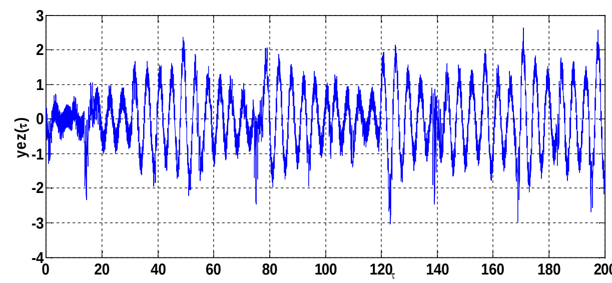
Fig. 7 Uniformly distributed random noise in the range [-1,1].



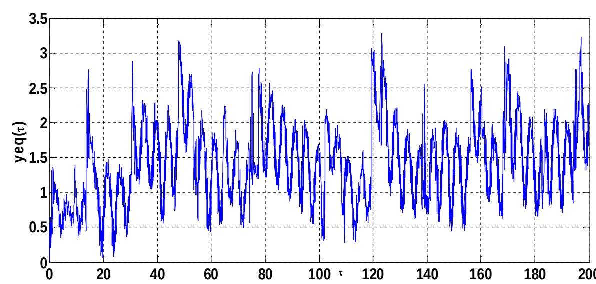
(a)



(b)

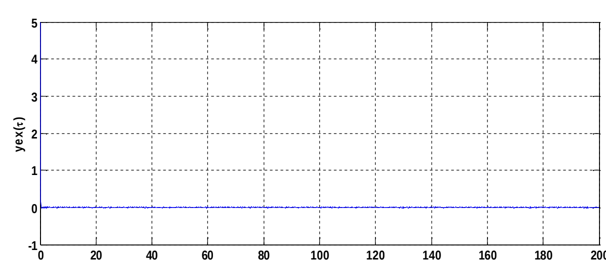


(c)

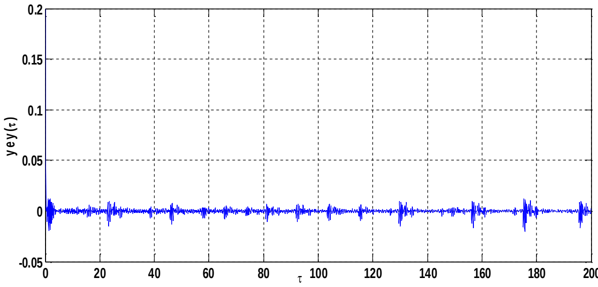


(d)

Fig. 8 The time history of the synchronization errors and error norm in presence of the disturbance when no control input is applied : (a): $y_{e_x}(\tau)$ **;(b):** $y_{e_y}(\tau)$ **(c):** $y_{e_z}(\tau)$ **(d):** $y_{e_q}(\tau)$



(a)



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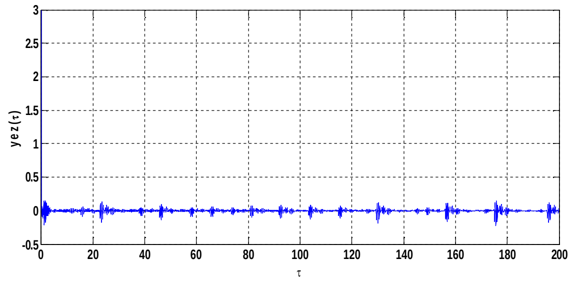
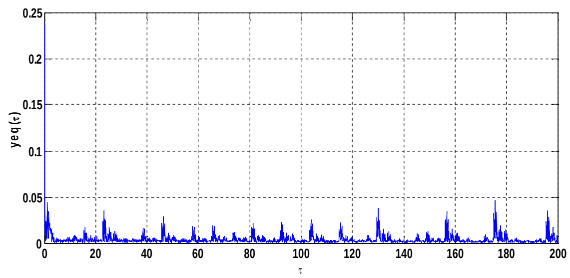


(c)



(d)

Fig. 9. Time evolution of the synchronization errors and error norm in presence of the disturbance; (a): $y_{e_x}(\tau)$; (b): $y_{e_y}(\tau)$; (c): $y_{e_z}(\tau)$ and (d): $y_{e_q}(\tau)$

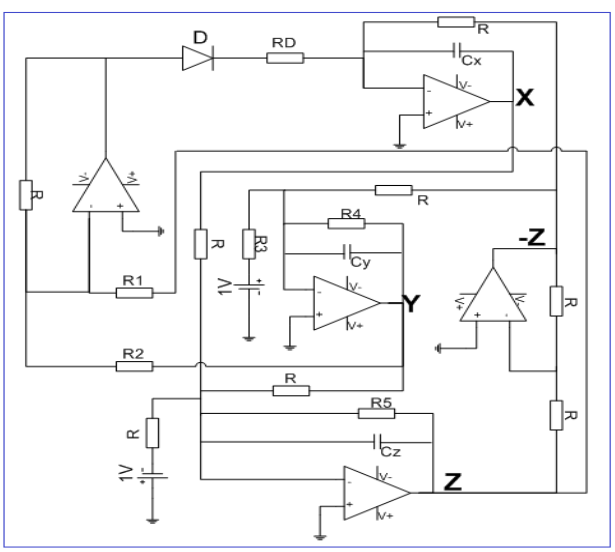
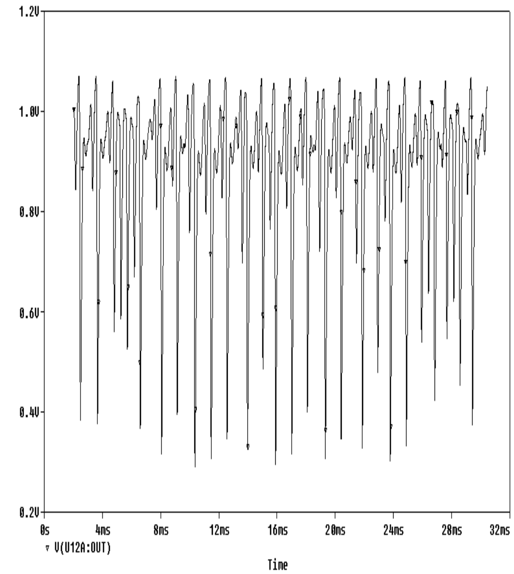
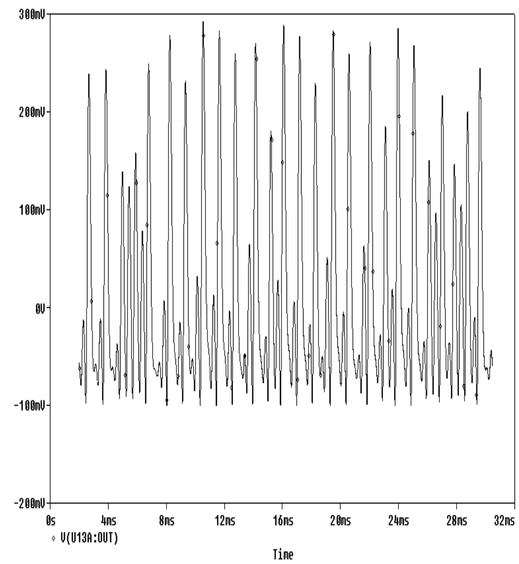
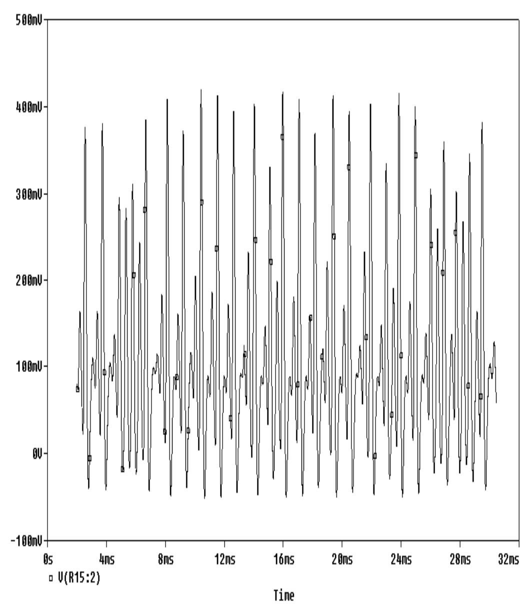


Fig. 10. Electrical analog model of MCO on Pspice







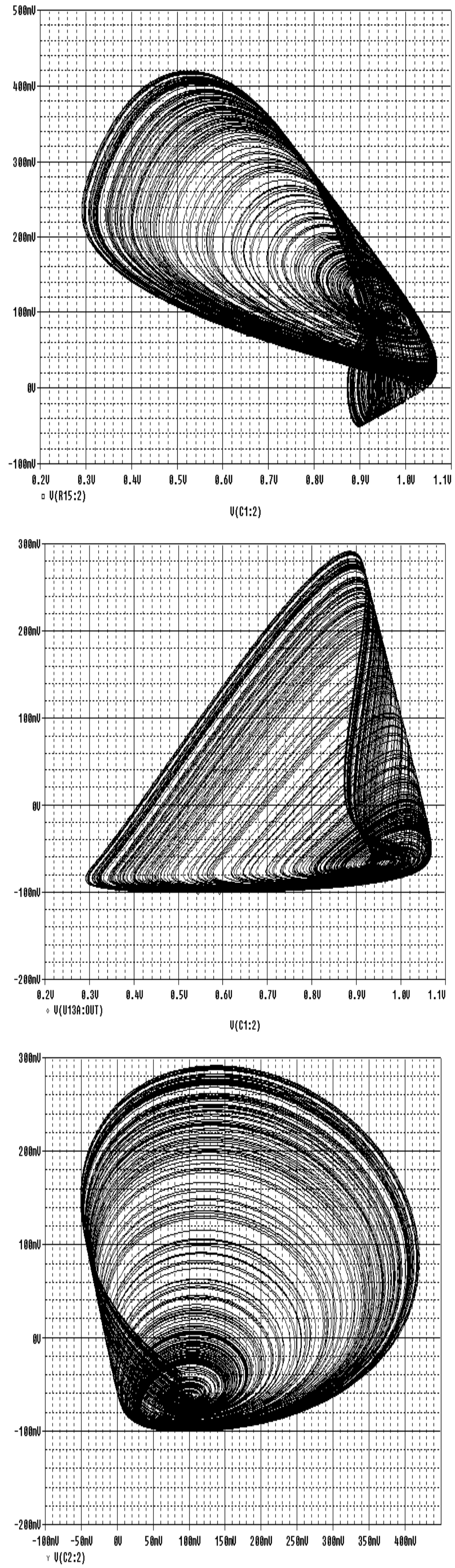


Fig. 11. Time history and phase portraits of the MCOOA

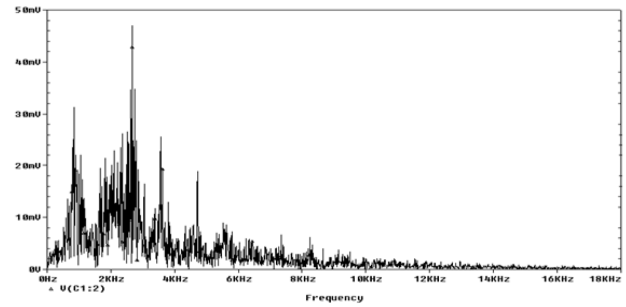


Fig. 12. Frequency spectrum of one output state of MCOOA

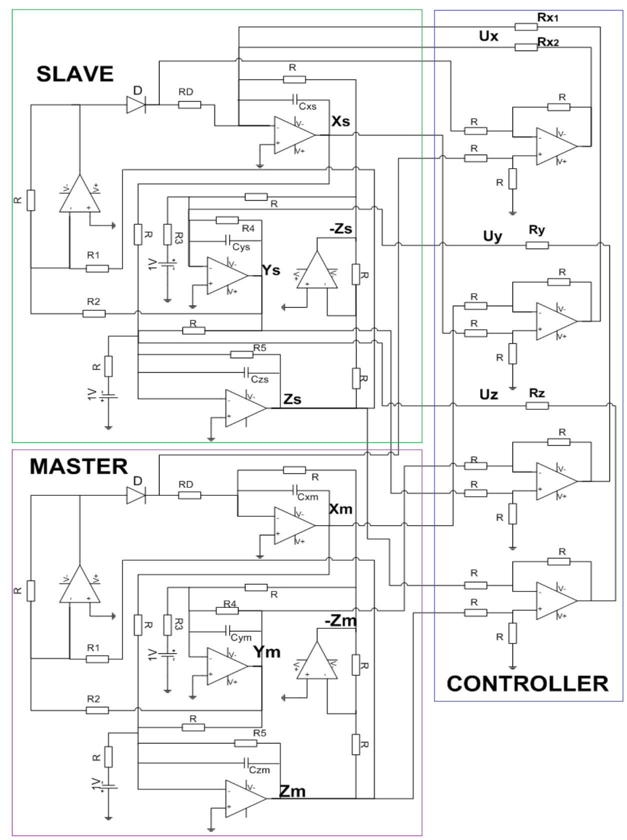


Fig. 13: Pspice implementation of the passivity-based synchronization process of MCOOA

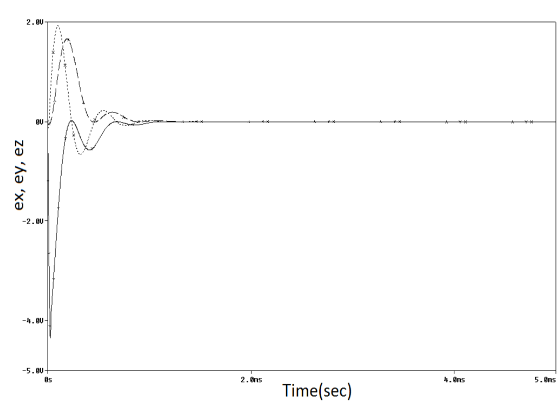


Fig. 14 Time variation of the synchronization errors from a Pspice simulation

$V_{CXm}(t)-V_{CXs}(t)$: continuous line; $V_{CYm}(t)-V_{CYs}(t)$: long dash line ; $V_{CZm}(t)-V_{CZs}(t)$: short dash line

Conclusion

In this paper, we proposed a passivity-based synchronization strategy of modified Colpitts oscillators with an external disturbance perturbation. Based on the Lyapunov theory and linear matrix inequality (LMI), A new sufficient condition is derived that can not only make the outputs of both master and slave systems reach H_∞ synchronization with the passage of time but also reduce the influence of the disturbance and guarantees asymptotic synchronization between the drive system and the response systems. Circuit simulations are investigated and show the feasibility of the synchronization scheme. As the simulation show, the proposed scheme could achieved passivity-based synchronization of two uncertain modified Colpitts oscillator and maintain robust stable synchronization in the presence of bounded external disturbances.

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