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RESEARCH ARTICLE

EXPECTED TIME TO RECRUITMENT IN SINGLE GRADED MANPOWER SYSTEM WHEN THRESHOLD HAS GAMMA DISTRIBUTION

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In large organizations the flow of individuals between the various ranks is a task which requires careful and detailed monitoring. Here to consider only single graded organization. The recruitment can be made by the organization itself or by some external agencies to avoid delay and huge overhead cost. The expected time to recruitment in the organization is obtained using shock model approach, assuming that the threshold follows gamma distribution. Numerical illustrations are provided for a better understanding of the model.

Key words: Recruitment, Shock model, Threshold, Wastage

INTRODUCTION

The main factors which determine the behavior of a manpower system are recruitment, promotion and wastage. The exit of personnel from an organization is a common phenomenon, which is known as wastage. There are several stochastic models dealing with wastage are found in Bartholomew and Forbes (1979). The organization goes for recruitments as and when the cumulative loss of manpower on successive occasions crosses a random threshold level beyond which the marketing activities would be affected. Esary, Marshall and Proschan (1973) discussed that any component or device when exposed to shock which cause damage to the device or system is likely to fail when the total accumulated of damage determines the life time of the component or device. A number of models derived for recruitment are discussed by Bartholomew (1971), Girnold and Marshall (1977). Using the shock model approach and cumulative damage process, the expected time to recruitment and its variance have been obtained by Sathiyamoorthi and Parthasarathy (2003). In this paper we made an attempt to determine the expected time to recruitment and its variance when threshold follows gamma distribution.

ASSUMPTION

1. Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives, and promotions are made.
2. The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
3. The process of depletion is linear and cumulative.
4. The inter arrival times between successive occasions of wastage are i.i.d. random variables.
5. If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable. The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

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NOTATIONS

X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i = 1,2,\dots,k$ and X_i's are i.i.d and $X_i = X$ for all i.

Y : a continuous random variable denoting the threshold level having Gamma distribution.

$g(.)$: The probability density functions of X.

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum X_i$

T : a continuous random variable denoting time to breakdown of the system

$g^*(.)$: Laplace transform of $g(.)$.

$g_k^*(.)$: Laplace transform of $g_k(.)$.

$h(.)$: The p.d.f. of random threshold level which has Gamma distribution and $H(.)$ is the corresponding c.d.f.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding c.d.f. $F(.)$.

$F_k(t)$: The k-fold convolution functions of $F(.)$.

$S(.)$: The survivor function i.e. $P\ [T > t]$.

$L(t)$: $1 - S(t)$

$V_k(t)$: Probability that there are exactly 'k' policies decisions in $(0, t]$.

RESULTS

One of the important families of distributions in lifetime tests is the gamma distribution. Two parameter gamma distribution is most popular distribution for analyzing life time data. It has several desirable properties see Johnson, Kotz and Balakrishnan (1994, 1995). Let Y be the random variable which follows gamma distribution with parameter λ and n $Y \sim G(\lambda, n)$

$$h(y) = \frac{e^{-\lambda y}\lambda^n y^{n-1}}{\Gamma n}\ ;\ \lambda > 0; n > 0$$

$$H(y) = \int_0^y h(y)\, dy$$

$$\overline{H(y)} = 1 - H(y)$$

$$\overline{H(y)} = \sum_{i=0}^{n-1} \frac{e^{-\lambda y}(\lambda y)^i}{\Gamma i}$$

Now,

$P(X_1 + X_2 + \cdots + X_k < Y) =$ P [the system does not fail, after k epochs of exits].

$$P\left[\sum X_i < Y\right] = \int_0^\infty g_k(x)\, \overline{H(x)}\, dx$$

$$= \int_0^\infty g_k(x) \left[\sum_{i=0}^{n-1} \frac{e^{-\lambda x}(\lambda x)^i}{\Gamma i}\right] dx$$

$$= \sum_{i=0}^{n-1} \frac{\lambda^i}{\Gamma i} \int_0^\infty g_k(x)\, e^{-\lambda x} x^i dx$$

$$= \sum_{i=0}^{n-1} \frac{\lambda^i}{\Gamma i} \left[\int_0^\infty g(x)\, e^{-\lambda x} x^i dx\right]^k$$

$$= \sum_{i=0}^{n-1} \left[\frac{\lambda^i}{\Gamma i} g^*(x^i \lambda)\right]^k$$

$$= \sum_{i=0}^{n-1} \left[\frac{\lambda^i}{\Gamma i} \frac{\Gamma i\, b}{(b+\lambda)^{i+1}}\right]^k \qquad \textit{see Appendix 1}$$

$$P\left[\sum_{i=0}^{n-1} X_i < Y\right] = \sum_{i=0}^{n-1} \left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^k \ or\ \left(\frac{b}{b+\lambda}\right)^k \sum_{i=0}^{n-1} \left[\left(\frac{\lambda}{b+\lambda}\right)^i\right]^k$$

The survival function $S(t)$ which is the probability that an individual survives for a time t

$S(t) = P(T > t) =$ Probability that the system survives beyond t

$$= \sum_{k=0}^{\infty} P\ \{\text{there are exactly k instants of exists in } (0,t] * P\ (\text{the system does not fail in } (0,t]\}$$

It is also known from renewal theory that

$$P(\text{exactly k policy decision in } (0,t]) = F_k(t) - F_{k+1}(t) \qquad \text{with } F_0(t) = 1$$

$$S(t) = \sum_{k=0}^{\infty} V_k(t)\, P\left[\sum X_i < Y\right]$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \sum_{i=0}^{n-1} \left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^k$$

$$S(t) = 1 - \sum_{i=0}^{n-1} \left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right] \sum_{k=1}^{\infty} F_k(t) \left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^{k-1} \quad \textit{see Appendix 2}$$

$$P(T < t) = L(T) = 1 - S(t)$$

$$L(T) = \sum_{i=0}^{n-1} \left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right] \sum_{k=1}^{\infty} F_k(t) \left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^{k-1}$$

Now, taking Laplace transform of $L(t)$, we get

$$L^*(s) = \sum_{i=0}^{n-1} \left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right] \sum_{k=1}^{\infty} f_k^*(s) \left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^{k-1}$$

$$= \sum_{i=0}^{n-1} \left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right] f^*(s) \sum_{k=1}^{\infty} \left[f^*(s) \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^{k-1}$$

$$= \sum_{i=0}^{n-1} \frac{\left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right] f^*(s)}{1 \quad \frac{b\lambda^i}{(b+\lambda)^{i+1}}\ f^*(s)}$$

Where $[f^*(s)]$ is Laplace transform of $F_k(t)$ since the inter arrival times are i.i.d.

Let $f(\cdot) \sim \exp(c);\ f^*(s) = \left(\frac{c}{c+s}\right)$

So, $$L^*(s) = \sum_{i=0}^{n-1} \frac{\left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right] \frac{c}{c+s}}{1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}} \frac{c}{c+s}}$$

$$\therefore L^*(s) = \sum_{i=0}^{n-1} \frac{[(b+\lambda)^{i+1} - b\lambda^i]c}{(b+\lambda)^{i+1}(c+s) - b\lambda^i c}$$

One can obtain expected time to recruitment $E(T)$ and variance $V(T)$ by using

$$E(T) = -\frac{d}{ds} L^*(s) \quad \text{given } s = 0$$

$$E(T^2) = \frac{d^2 L^*(S)}{ds^2}$$

From which $V(T)$ can be obtained.

$$\frac{d}{ds} L^*(s) = -\sum_{i=0}^{n-1} -\frac{[(b+\lambda)^{i+1} - b\lambda^i](b+\lambda)^{i+1} c}{[(b+\lambda)^{i+1}(c+s) - b\lambda^i c]^2}$$

$$= \sum_{i=0}^{n-1} \frac{c[(b+\lambda)^{i+1} - b\lambda^i](b+\lambda)^{i+1}}{c^2[(b+\lambda)^{i+1} - b\lambda^i]^2}$$

$$\therefore E(T) = \frac{1}{c} \sum_{i=0}^{n-1} \frac{(b+\lambda)^{i+1}}{(b+\lambda)^{i+1} - b\lambda^i}$$

$$E(T^2) = \frac{d}{ds} L^*(S) = -\sum_{i=0}^{n-1} \frac{[(b+\lambda)^{i+1} - b\lambda^i](b+\lambda)^{i+1}(c)}{[(b+\lambda)^{i+1}(c+s) - (b\lambda^i)c]^2}$$

$$\frac{d}{ds}\left[\frac{d}{ds} L^*(S)\right] = -\sum_{i=0}^{n-1} \frac{[(b+\lambda)^{i+1} - b\lambda^i](c)(-1)[(b+\lambda)^{i+1}(c+s) - (b\lambda^i)c](b+\lambda)^{i+1}}{[(b+\lambda)^{i+1}(c+s) - (b\lambda^i)c]^4}$$

$$\frac{d^2}{ds^2} L^*(S) = \sum_{i=0}^{n-1} \frac{2c[(b+\lambda)^{i+1} - b\lambda^i][(b+\lambda)^{i+1}(c+s) - (b\lambda^i)c][(b+\lambda)^{i+1}]}{[(b+\lambda)^{i+1}(c+s) - (b\lambda^i)c]^4} \quad \text{given } s = 0$$

$$= \sum_{i=0}^{n-1} \frac{2c[(b+\lambda)^{i+1} - b\lambda^i]c[(b+\lambda)^{i+1}]}{c^4[(b+\lambda)^{i+1} - (b\lambda^i)]^4}$$

$$E(T^2) = \frac{2}{c^2} \sum_{i=0}^{n-1} \left[\frac{(b+\lambda)^{i+1}}{(b+\lambda)^{i+1} - b\lambda^i}\right]^2$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$V(T) = \left[\frac{2}{c^2} \sum_{i=0}^{n-1} \left[\frac{(b+\lambda)^{i+1}}{(b+\lambda)^{i+1} - b\lambda^i}\right]^2\right] - \left[\frac{1}{c} \sum_{i=0}^{n-1} \frac{(b+\lambda)^{i+1}}{(b+\lambda)^{i+1} - b\lambda^i}\right]^2$$

NUMERICAL ILLUSTRATION

The tables and figures observe the time to recruitment and its variance for the total loss of manpower. For $E(T)$ and $V(T)$ when the threshold distribution has gamma with fixed parameter in different time period as

i) $\lambda = 0.4$ and $b = 0.2$ ii $\lambda = 0.8$ and $b = 0.2$ iii) $\lambda = 0.4$ and $b = 0.4$

For (i), (ii) and (iii) the inter-arrival time varies $c = 1,2, \dots .10$ and n varies $2, 4, 6, 8,$ *and* 10.

Table 1.1 $\lambda = 0.4, b = 0.2$

C	n=2	n=4	n=6	n=8	n=10
1	2.786	5.069	7.186	9.236	11.258
2	1.393	2.535	3.593	4.618	5.629
3	0.929	1.69	2.395	3.079	3.753
4	0.696	1.267	1.796	2.309	2.814
5	0.557	1.014	1.437	1.847	2.252
6	0.464	0.845	1.198	1.539	1.876
7	0.398	0.724	1.027	1.319	1.608
8	0.348	0.634	0.898	1.154	1.407
9	0.31	0.563	0.798	1.026	1.251
10	0.279	0.507	0.719	0.924	1.126

C	n=2	n=4	n=6	n=8	n=10
1	2.44	4.701	6.861	8.96	11.022
2	1.22	2.351	3.43	4.48	5.511
3	0.813	1.567	2.287	2.987	3.674
4	0.61	1.175	1.715	2.24	2.756
5	0.488	0.94	1.372	1.792	2.204
6	0.407	0.784	1.143	1.493	1.837
7	0.349	0.672	0.98	1.28	1.575
8	0.305	0.588	0.858	1.12	1.378
9	0.271	0.522	0.762	0.996	1.225
10	0.244	0.47	0.686	0.896	1.102

Table 1.3 $\lambda = 0.4, b = 0.4$

C	n=2	n=4	n=6	n=8	n=10
1	3.333	5.543	7.591	9.603	11.606
2	1.667	2.771	3.795	4.801	5.803
3	1.111	1.848	2.53	3.201	3.869
4	0.833	1.386	1.898	2.401	2.901
5	0.667	1.109	1.518	1.921	2.321
6	0.556	0.924	1.265	1.6	1.934
7	0.476	0.792	1.084	1.372	1.658
8	0.417	0.693	0.949	1.2	1.451
9	0.37	0.616	0.843	1.067	1.29
10	0.333	0.554	0.759	0.96	1.161

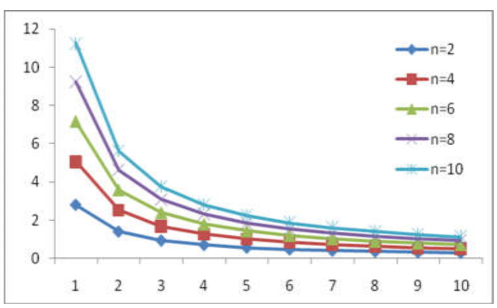

Figure 1.1

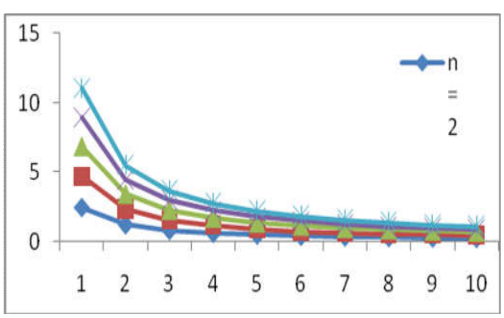

Figure 1.2

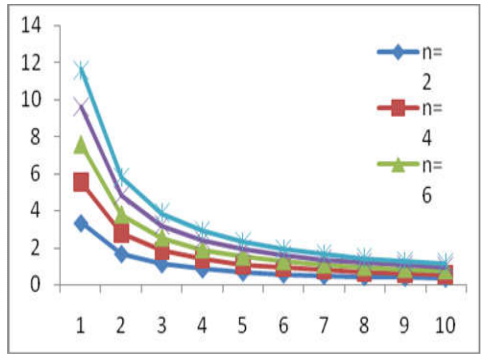

Figure 1.3

Table 2.1 $\lambda = 0.4, b = 0.2$

C	n=2	n=4	n=6	n=8	n=10
1	7.76	25.697	51.633	85.297	126.733
2	1.94	6.424	12.908	21.324	31.683
3	0.862	2.855	5.737	9.477	14.081
4	0.485	1.606	3.227	5.331	7.921
5	0.31	1.028	2.065	3.412	5.069
6	0.216	0.714	1.434	2.369	3.52
7	0.158	0.524	1.054	1.741	2.586
8	0.121	0.402	0.807	1.333	1.98
9	0.096	0.317	0.637	1.053	1.565
10	0.078	0.257	0.516	0.853	1.267

Table 2.2 $\lambda = 0.8, b = 0.2$

C	n=2	n=4	n=6	n=8	n=10
1	5.956	22.103	47.069	80.278	121.487
2	1.489	5.526	11.767	20.07	30.372
3	0.662	2.456	5.23	8.92	13.499
4	0.372	1.381	2.942	5.017	7.593
5	0.238	0.884	1.883	3.211	4.859
6	0.165	0.614	1.307	2.23	3.375
7	0.122	0.451	0.961	1.638	2.479
8	0.093	0.345	0.735	1.254	1.898
9	0.074	0.273	0.581	0.991	1.5
10	0.06	0.221	0.471	0.803	1.215

Table 2.3 $\lambda = 0.4, b = 0.4$

C	n=2	n=4	n=6	n=8	n=10
1	11.111	30.723	57.623	92.213	134.693
2	2.778	7.681	14.406	23.053	33.673
3	1.235	3.414	6.403	10.246	14.966
4	0.694	1.92	3.601	5.763	8.418
5	0.444	1.229	2.305	3.689	5.388
6	0.309	0.853	1.601	2.561	3.741
7	0.227	0.627	1.176	1.882	2.749
8	0.174	0.48	0.9	1.441	2.105
9	0.137	0.379	0.711	1.138	1.663
10	0.111	0.307	0.576	0.922	1.347

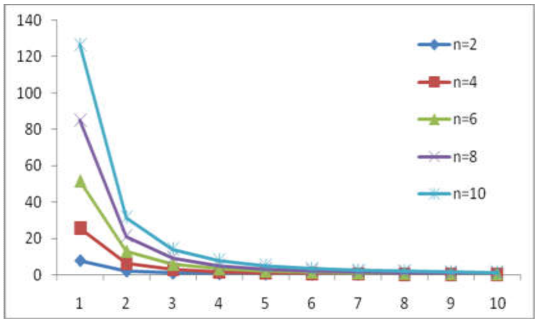

Figure 2.1

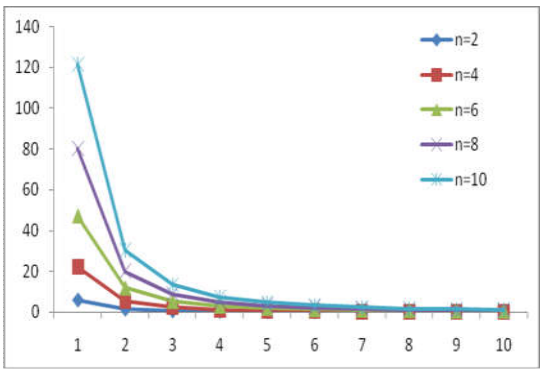

Figure 2.2

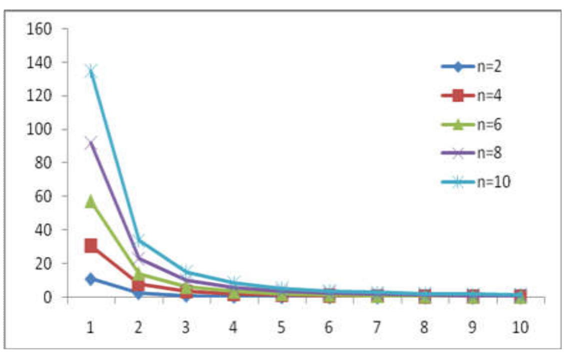

Figure 2.3

CONCLUSION

We come to the conclusion if the completed length of service increases (i.e., number of people leaving the job) the time to recruitment in an organization decreases. When the shape parameter increases the expected time to recruitment also increases. While fixing λ and b. So, we observed that when threshold follows gamma distribution which prolong the recruitment period. In all the cases when inter-arrival increase the time to recruitment decreases.

Appendix 1:

$$g^*(x^i\lambda) = (-1)^i \frac{d^i}{d\lambda^i}[L(be^{-bt})]$$

$$= (-1)^i \frac{d^i}{d\lambda^i}\left[\frac{b}{b+\lambda}\right]^i$$

$$= (-1)(-1)^i \frac{\Gamma ib}{(b+\lambda)^{i+1}}$$

$$= \frac{(-1)^{2i}\Gamma ib}{(b+\lambda)^{i+1}}$$

$$\therefore g^*(x^i\lambda) = \frac{\Gamma ib}{(b+\lambda)^{i+1}}$$

Appendix 2:

$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \sum_{i=0}^{n-1}\left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^k$$

$$= [F_0(t) - F_1(t)] + [F_1(t) - F_2(t)]\sum_{i=0}^{n-1}\frac{b\lambda^i}{(b+\lambda)^{i+1}} + [F_2(t) - F_3(t)]\sum_{i=0}^{n-1}\left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^2 + \cdots$$

$$S(T) = 1 - \left[1 - \sum_{i=0}^{n-1}\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]F_1(t) - \left[1 - \sum_{i=0}^{n-1}\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]F_2(t)\sum_{i=0}^{n-1}\frac{b\lambda^i}{(b+\lambda)^{i+1}} - \left[1 - \sum_{i=0}^{n-1}\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]F_3(t)\sum_{i=0}^{n-1}\left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^2 \cdots$$

$$= 1 - \left[1 - \sum_{i=0}^{n-1}\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]\sum_{k=1}^{\infty}F_k(t)\sum_{i=0}^{n-1}\left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^{k-1}$$

$$S(T) = 1 - \sum_{i=0}^{n-1}\left[1 - \frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]\sum_{i=0}^{n-1}F_k(t)\left[\frac{b\lambda^i}{(b+\lambda)^{i+1}}\right]^{k-1}$$

REFERENCES

Barthlomew, D.J., 1971. The Statistical Approach to Manpower Planning. Statistician, 20: 3-26.

Bartholomew, D.J., and A.F Forbes, 1979. Statistical Techniques for Manpower Planning. John Wiley and Sons.

Esary, J.D., Marshall, A.W. and Proschan, F. 1973. Shock models and wear processes. Ann. Probability, 1(4): 627-649.

Grinold, R.C. and Marshall, K.J. 1977. Manpower planning Models, North Holland, New York.

Johnson, Kotz and Balakrishnan, 1994. Continuous Univariate Distributions-1. 2nd edition, John Wiley & Sons, New York.

Johnson, Kotz and Balakrishnan, 1994. Continuous Univariate Distributions-2. 2nd edition, John Wiley & Sons, New York.

Sathiyamoorthi, R. and S. Parthasarathy, 2003. On the expected time to recruitment when threshold distribution has SCBZ property. IJMS, 19(3): 233-240

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