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RESEARCH ARTICLE

SUPERFLUID TO MOTT INSULATOR TRANSITION OF VORTEX DRIVEN 1D BOSE- HUBBARD MODEL

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ABSTRACT

The aim of this work is to probe observable effects of vortex driven Superfluid-Mott insulator transition using the rotational Bose-Hubbard mode kept inhomogeneous by a Berry curvature, considering neutral bosons confined in a 1D confinement about the x-direction. The 1D model includes two-or-three-body onsite interaction in order to probe further into the interactions. Our theoretical results show notable effects. These effects include vortex escape, ground state degeneracy showing some symmetry breaking due to vortex nucleation, higher degenerate excited states and preferential accommodation of vortex energy. To validate our results, the critical values for the Superfluid to Mott-insulator transition for the 2 bosons $U_c/t = 2.00$, in agreement with Manuela et. al.(2004) where the critical value is $U_c/t \approx 2.10 \pm 0.1$ in 1D Mott-insulator transition and also as calculated in Batrouni et al (1990), while in the 3 bosons interaction case the transition point is $U_c/t = 3.20$ closely equal to $U_c/t \approx 3.40$ obtained by Jaksch et al. (1998), Kunher and Moniem (1998), Pai and Pandit (2005). We have discussed these notable effects extensively and conclude from our results that the vortex has no significant effect on the on the 1D Superfluid to Mott insulation transition points, but has much significant effect on the energy spectra excitation and degeneracy . We hope to employ our result as feasible candidate for implementation of the Dirac's principle of superposition of evolution of time proposed by Ibeh and Akpojortor (2020) in probing the problem of "negative time" in quantum mechanics.

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INTRODUCTION

Ultracold atoms have temperatures near zero Kelvin and are used to study quantum mechanical properties of atoms. The atoms are first trapped, precooled and then lacer cooled using magneto-optical traps. Ultra-cold atoms are used in the study of several phenomena including Bose Einstein condensation, quantum phase transitions, bosonic superfluid, many body spin dynamics and BEC-BCS crossover. The most interesting quantum gas of bosons include the Mott-insulating phase and a Superfluid phase. The Mott-insulator phase first discussed by Fisher and his co-workers (1989), corresponds to an average integer occupation of the lattice sites and is accompanied by a loss of compressibility, and that this Mott-insulator phase contains a Superfluid phase, indicating a phase transition between the phases. Superfluidity, first discovered in 1937 by Pyotr Kapitza, is a delocalized state of non-zero condensed fraction and gapless energy spectrum Fabrice (2018). The Superfluid phase includes atoms spreading out over the entire lattice, while the Mott insulator phase involves the localized at individual lattice sites. The addition of a Superfluid phase to a Mott insulator phase in an interacting gas of bosons shows a quantum phase transition between these phases. Triggered by experimental observation in a seminar by Greiner et al. (2002), several studies on Superfluid-Mott insulator (SF-MI) transition of bosonic particles have populated condensed matter Physics research community, including those reported in references that follow. Sshunji (2004) investigated the dynamics of SF-MI transition of spin-bosons by studying collective excitation of bosons condensates in a shallow optical lattices, within the framework of the Bose-Hubbard model. Batrouni et. al.(2002) studied the effect of the trap potential on the SF-MI transition in Bose-Hubbard model and observed very different behaviour where the Mott transition does not occur via a traditional quantum phase transition. Jacksch *et. al*. (1998) studied SF-MI quantum phase transition of bosons in optical lattice with super imposed harmonic trap. Thonhauser *et. al.* (2007) observed the SF-MI transition for double well trap potential showing many characteristic features. In Wu et al. (2004), single vortex structure was considered for strongly interacting bosons in rotating optical lattice and studied the nature of the vortex core near the SF-MI transition within the mean field theory of Bose-Hubbard model. Near the SF-MI transition, studied several sudden and unexpected structural phase transitions due to the pinning of the vortices by optical lattice. Ippei and Anatoli (2011) determined the critical point of the SF-MI transition for arbitrary integer filling factors in 1D. Stoof et al. (2009) considered the impossibility of finding a transition from the Superfluid to the Mott-insulating state from the Bogoliubov's theory for a weakly interacting Bose gas, and have employed mean field theory in describing

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the transition between a normal gas and the Superfluid state. The quantum phase transition for Superfluid limit $\frac{U}{t} \to 0$ and the Mott-insulator limit $\frac{t}{U} \to 0$ where considered as completely different phase of matter for the two limiting cases Fabrice (2018) calculated the critical value of U for which SF-MI transition occurs. Sengupta and Dupuis (2005) studied MI-SF transition in Bose gas using a strong coupling approach. In Manuela et al. (2018), the SF-MI transition was studied for 1,2 and 3D geometries. Non-linear phenomena in BEC has become a subject of interest in condensed matter physics and vortices have been presented as fundamental non-linear excitation of BEC by Christophe and Yves (2001). For instance, the interacting BEC is a Superfluid whose motion is constrained by irrotation and when it carries angular momentum formation of quantum vortices is triggered. Also, condensate energy and vortex have interesting influence on one another. The energy spectrum outside the transition phase plays vital role in the vortex formation of the BEC and the appearance of vortex states could predict the economy of the condensate energy spectrum. Vortices as topological defects of ultracold atoms is used in the simulation of formation of new materials structures and derivations of more complex physical properties via interactions and self organization. Recently, Emil and Tapios (2023) predicted the existence of a vortex spin in a Superfluid, which has a close analogy with gravitational field theory.The Superfluids order parameters have non-vanishing spin or orbital angular momentum. Motivated by the predictions in Emil and Tapios (2023), this paper seeks to probe into the mechanism of vortex in Superfluids considering that the Superfluid order parameters as having non-vanishing spin angular momentum. This paper is planned as follows. In section 2, we introduce the Bose-Hubbard model with 1-dimensional vortex-potential kept inhomogeneous by a Berry curvature in the x-direction. The 1D model includes two-or-three-body onsite interaction in order to probe further into the effects of bosonic interactions. In section 2.1, we simulate the Bose-Hubbard Hamiltonian on two-bosons with two-body onsite interaction. This will include the Superfluid (SF) phase in the limit of small kinetic energy t ➔ 0 and the Mott-insulator (MI) phase in the limit of small potential energy $u \to 0$. In the SF and MI phases, the ground state energy and excited states energies are derived by solving the eigenvalue determinants in the BEC thermodynamic limit $T < T_C$ (T_c is transition temperature of BEC). Following the analysis in section 2.1, in section 2.2 we then study SF-MI transition for three interacting bosons on three sites of neutral bosons confined in a 1D trap which rotates about the x-direction, considering standard single particle hopping and pair hopping. In section 3, we give a summary of our results and this will be followed by a conclusion.

Model Formulation: We consider neutral bosons confined in a 1D magneto-optical trap which rotates about the x-axis. We investigate the interacting bosons considering effective two- and three-body onsite interactions. Following Mukesh and Bishwajyoti (2019) and Bhat et al.(2006), the 1D Bose-Hubbard model with vortex can be written as

$$H = -[(t+\gamma)\sum_{\langle i,j\rangle}(C_i^{+}C_j + H.C)] + U_{nn}\sum_{\langle i,j\rangle} n_i n_i + H_{OI} \qquad \text{.............................(2.1)}$$

Where $C_i^{+}(C_j)$ creates (annihilates) a boson at site i, $n_i = C_i^{+}C_j$ gives number of bosons at site i. Parameters t, γ and U_{nn} are the strengths of the hopping, vorticity and onsite interactions respectively. The last term, H_{OI} is Hamiltonian describing two-body on-site interaction or three body onsite interaction. Following Mukesh and Bishwajyoti (2019) we define $\gamma = \frac{i\hbar\Omega Cos\,\theta_{IJ}}{2x_{IJ}^{2}}$. The Berry curvature $\frac{1}{2x_{IJ}^{2}}$ plays the role of keeping the system inhomogeneous, moreover realizing Bose-Einstein condensation is impossible in 1D or 2D homogeneous systems. Mermin and Wagner (1966), and Hohenberg (1967), but does occur in atom traps because the confining potential modifies the density of states Petrov et al. (2004).

The Hamiltonian, H_{OI}, in Equ.(2.1) is popularly given by

$$H_{OI} = \begin{cases} \frac{U}{2} n_i(n_i - 1), two-body \\ \frac{U}{6} n_i(n_i - 1)(n_i - 2), three-body \end{cases} \qquad (2.2)$$

Fig. 1. 1D sites of interacting bosons accommodating 1,2 or 3 bosons per site due to hopping

2.1 Two-Bosons With Onsite Two-Body Interaction

Substituting the two-body onsite interaction in Equ. (2.2) into the Hamiltonian (2.1), the matrix elements for the Hamiltonian acting on the state $|1\rangle$ and $|2\rangle$, can be obtained.

First, we define the thermodynamics limit $T < T_c$ following Einstein's prediction in 1925 that many bosons will prefer to occupy a single particle quantum state – a condensed matter phenomenon known as Bose-Einstein condensation,

Employing the Hamiltonian on state $|1\rangle$ and $|2\rangle$,

$$H_{u/2}|1\rangle = \frac{u}{2}|1\rangle, H_{(t+\gamma)}|2\rangle = -(t+\gamma)|1\rangle, H_{(t+\gamma)}|1\rangle = 0, H_{u_{nn}}|2 = u|2\rangle\rangle$$

$$H^{2B} = \begin{pmatrix} \frac{U}{2} & 0 \\ -(t+\gamma) & U \end{pmatrix} \qquad (2.3)$$

To probe the SF phase and MI phase, we transform the Hamiltonian (2.3) by introducing dimensionless parameters $u/t\,(t \to 0)$ and $t/u\,(u \to 0)$ respectively. We solve the system as eigenvalue problem and obtain the energy spectrum.

The eigenvalue problem becomes

$$\begin{vmatrix} (\frac{u}{2t}-\frac{E}{t}) & 0 \\ -(1+\frac{\gamma}{t}) & (\frac{u}{t}-\frac{E}{t}) \end{vmatrix} = 0 \quad (2.4)$$

The ground state and the first excited state energy are, respectively,

$$\frac{E_g^{2B(SF)}}{t} = \frac{U}{2t} \quad (2.5a)$$

$$\frac{E_1^{2B(SF)}}{t} = \frac{U}{t} \quad (2.5b)$$

From the Hamiltonian (2.3), the Mott-insulator eigenvalue equation can be obtained

$$\begin{vmatrix} (\frac{1}{2}-\frac{E}{u}) & 0 \\ -(\frac{t+u}{u}) & (1-\frac{E}{u}) \end{vmatrix} = 0 \quad (2.6)$$

We have the energy the ground and first excited energy spectrum, respectively,

$$\frac{E_g^{2B(MI)}}{U} = 1 \quad (2.7a)$$

$$\frac{E_1^{2B(MI)}}{U} = 2 \quad (2.7b)$$

Three-Bosons With Single Particle Hopping: We simply substitute the three-body onsite interaction, Equ.(2.2), into the Bose-Hubbard model, Equ.(2.1), we consider interaction for standard single particle hopping. Table 1. Matrix elements for the Hamiltonian with single particle hopping: . $\gamma = \frac{i\hbar\Omega Cos\,\theta_{IJ}}{2x_{IJ}^2}$.

Table 1. Matrix formulation data for Superfluid-Mott-insulator transition in the case of three-bosons with single particle hopping

State	$H_{(t+\gamma)-matrixeelement}$	$H_{U_{nn}} - matrixelements$	$H_{U/6} - matrixelements$
$\|1\rangle$	0	0	$\frac{U}{6}\|1\rangle$
$\|2\rangle$	0	0	$\frac{U}{6}\|2\rangle$
$\|3\rangle$	0	0	$\frac{U}{6}\|3\rangle$
$\|4\rangle$	$-(t+\gamma)\ \|1\rangle$	$U\|4\rangle$	0
$\|5\rangle$	$-(t+\gamma)\ \|2\rangle$	$U\|5\rangle$	0
$\|6\rangle$	$-(t+\gamma)\ \|4\rangle+\|5\rangle$	$U\|6\rangle$	0

Table 1 provides us with data for Superfluid-Mott-insulator transition in this case. The Hamiltonian matrix is

$$H_{SPH}^{3B} = \begin{pmatrix} \frac{U}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{U}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{U}{6} & 0 & 0 & 0 \\ y & 0 & 0 & U & 0 & 0 \\ 0 & y & 0 & 0 & U & 0 \\ 0 & 0 & 0 & y & y & U \end{pmatrix} \quad (2.8)$$

Where $y = -(t+\gamma)$ .Probing, the Superfluid phase energy spectrum, we obtain the eigenvalues of the Hamiltonian (2.8) in the limit $U/_t\,(t \to 0)$.

Th

$$\begin{vmatrix} x & 0 & 0 & 0 & 0 & 0 \\ 0 & x & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \\ q & 0 & 0 & z & 0 & 0 \\ 0 & q & 0 & 0 & z & 0 \\ 0 & 0 & 0 & q & q & z \end{vmatrix} = 0 \tag{2.9}$$

In Equ.(2.9), we have used $x=\left(\frac{E}{t}-\frac{U}{6t}\right), z=\left(\frac{E}{t}-\frac{U}{t}\right), q=\left(1+\frac{\gamma}{t}\right)$. Solving the lower triangular determinant, we obtain the Superfluid double fold degenerate ground state, the triple fold degenerate first excited state and the non-degenerate second excited state are respectively obtained as follows:

$$\frac{E^{3B(SF)}_{g(SPH)}}{t} = \frac{(U-\gamma)}{t} - 1 \tag{2.10a}$$

$$\frac{E^{3B(SF)}_{1(SPH)}}{t} = \frac{U}{6t} \tag{2.10b}$$

$$\frac{E^{3B(SF)}_{2(SPH)}}{t} = \frac{2U}{3t}, \tag{2.10c}$$

For the Mott-insulator phase, the Hamiltonian Equ.(2.8) is yet transformed in the limit

$t/U\ (t \to 0)$ and the eigenvalue problem solved using the well known linear algebra method.

Putting the substitutions $A=\left(\frac{E}{U}-\frac{1}{6}\right), B=\left(\frac{E}{U}-1\right), C=\frac{(t+\gamma)}{U}$ we obtain the problem and the Characteristics determinant

$$\begin{vmatrix} A & 0 & 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & 0 \\ 0 & 0 & A & 0 & 0 & 0 \\ C & 0 & 0 & B & 0 & 0 \\ 0 & C & 0 & 0 & B & 0 \\ 0 & 0 & 0 & C & C & B \end{vmatrix} = 0 \tag{2.11}$$

And the eigenvalues; The double fold degenerate ground state, the triple fold degenerate first excited state and the non-degenerate second excited state are respectively:

$$\frac{E^{3B(MI)}_{g(SPH)}}{U} = 1 - \frac{(t+\gamma)}{U} \tag{2.12a}$$

$$\frac{E^{3B(MI)}_{1(SPH)}}{U} = \frac{1}{6} \tag{2.12b}$$

$$\frac{E^{3B(MI)}_{2(SPH)}}{U} = 1 \tag{2.12c}$$

2.3 Three-Bosons With Pair Hopping

Table 2. The matrix elements of 1D Bose-Hubbard Hamiltonian for three bosons interacting with pair hopping

State	$H_{(t+\gamma)-matrixelement}$	$H_{U-matrixelements}$	$H_{U/6}-matrixelements$
$\|1\rangle$	0	0	$\frac{U}{6}\|1\rangle$
$\|2\rangle$	0	0	$\frac{U}{6}\|2\rangle$
$\|3\rangle$	0	0	$\frac{U}{6}\|3\rangle$
$\|4\rangle$	$-(t+\gamma)\ \|2\rangle$	$U\|4\rangle$	0
$\|5\rangle$	$-(t+\gamma)\ \|3\rangle$	$U\|5\rangle$	0
$\|6\rangle$	0	$U\|6\rangle$	0

The Hamiltonian matrix in this case can simply be obtained from Table 2

$$H_{PH}^{3B} = \begin{pmatrix} \frac{U}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{U}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{U}{2} & 0 & 0 & 0 \\ 0 & y & 0 & U & 0 & 0 \\ 0 & 0 & y & 0 & U & 0 \\ 0 & 0 & 0 & 0 & 0 & U \end{pmatrix} \tag{2.13}$$

The Hamiltonian (2.13) yields the Superfluid eigenvalue determinant

$$\begin{vmatrix} x & 0 & 0 & 0 & 0 & 0 \\ 0 & x & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \\ q & 0 & 0 & z & 0 & 0 \\ 0 & q & 0 & 0 & z & 0 \\ 0 & 0 & 0 & q & q & z \end{vmatrix} = 0 \tag{2.14}$$

We then calculate the energy spectrum for the Superfluid. The triple fold degenerate ground state, the double fold degenerate first excited state, and non-degenerate second excited state.

$$\frac{E_{g(PH)}^{3B(SF)}}{t} = \frac{U}{6t} \tag{2.15a}$$

$$\frac{E_{1(PH)}^{3B(SF)}}{t} = \frac{U}{t} - (t+\gamma) \tag{2.15b}$$

$$\frac{E_{2(PH)}^{3B(SF)}}{t} = \frac{U}{t}. \tag{2.15c}$$

In the Mott-insulator phase, the eigenvalue determinant becomes

$$\begin{vmatrix} A & 0 & 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & 0 \\ 0 & 0 & A & 0 & 0 & 0 \\ 0 & 0 & 0 & B & 0 & 0 \\ 0 & C & 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 & 0 & B \end{vmatrix} = 0 \tag{2.16}$$

$$\frac{E_{g(PH)}^{3B(MI)}}{U} = \frac{1}{6} \tag{2.17a}$$

$$\frac{E_{1(PH)}^{3B(MI)}}{U} = 1 - \frac{(t+\gamma)}{U} \tag{2.17b}$$

$$\frac{E_{2(PH)}^{3B(MI)}}{U} = 1 \tag{2.17c}$$

RESULTS AND DISCUSSION

RESULTS

Table 3. Energy spectrum profiles of SF and MI phases for the 2 interacting bosons

U/t	SF Energy spectrum		MI Energy spectrum		SF-MI transition
	$\frac{E_g^{SF}}{t}$	$\frac{E_1^{SF}}{t}$	$\frac{E_g^{MI}}{U}$	$\frac{E_1^{MI}}{U}$	ΔE_g
1.40	0.70	1.40	1.00	2.00	-0.30
1.60	0.80	1.60	1.00	2.00	-0.20
1.80	0.90	1.80	1.00	2.00	-0.10
2.00	**1.00**	**2.00**	**1.00**	**2.00**	**0.00**
2.20	1.10	2.20	1.00	2.00	0.10
2.40	1.20	2.40	1.00	2.00	0.20
2.60	1.30	2.60	1.00	2.00	0.30
2.80	1.40	2.80	1.00	2.00	0.40
3.00	1.50	3.00	1.00	2.00	0.50
3.20	1.60	3.20	1.00	2.00	0.60

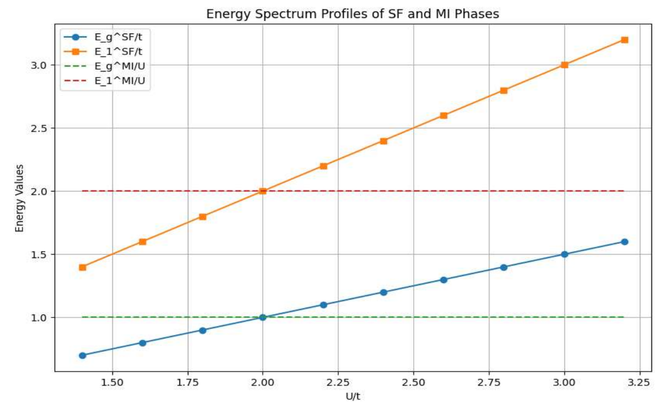

Fig.2: Energy spectrum for the 2 bosons interaction in the Superfluid (SF) and Mott-insulator (MI) phases at varying values of U/t and of t/U with repulsive vorticity. The blue full line is values for the SF ground state, the yellow full line is values for the SF first excited energy, the green broken line is values for the MI ground state and the purple broken line for the first excited energy of the MI phase.

Table 4. Energy spectrum profiles of SF and MI phases for the 3 interacting bosons with single particle hopping

U/t	γ/t ($\times i\hbar$)	SF Energy spectrum			MI Energy spectrum			SF-MI transition
t/U	γ/U ($\times i\hbar$)	$\frac{E_g^{SF}}{t}$	$\frac{E_1^{SF}}{t}$	$\frac{E_2^{SF}}{t}$	$\frac{E_g^{MI}}{U}$	$\frac{E_1^{MI}}{U}$	$\frac{E_2^{MI}}{U}$	ΔE_g
1.40	0.20	-(0.90 + 0.20 $i\hbar$)	0.02	0.07	0.90 - 0.20 $i\hbar$	0.17	1.00	-1.80
1.60	0.40	-(0.80 + 0.40 $i\hbar$)	0.03	0.13	0.80 - 0.40 $i\hbar$	0.17	1.00	-1.60
1.80	0.60	-(0.70 + 0.60 $i\hbar$)	0.01	0.20	0.70 - 0.60 $i\hbar$	0.17	1.00	-1.40
2.00	0.80	-(0.60 + 0.80 $i\hbar$)	0.07	0.27	0.60 - 0.80 $i\hbar$	0.17	1.00	-1.20
2.20	1.00	-(0.50 + 1.00 $i\hbar$)	0.08	0.33	0.50 - 1.00 $i\hbar$	0.17	1.00	-1.00
2.40	1.20	-(0.40 + 1.20 $i\hbar$)	0.10	0.40	0.40 - 1.20 $i\hbar$	0.17	1.00	-0.80
2.60	1.40	-(0.30 + 1.40 $i\hbar$)	0.12	0.47	0.30 - 1.40 $i\hbar$	0.17	1.00	-0.60
2.80	1.60	-(0.20 + 1.60 $i\hbar$)	0.13	0.53	0.20 - 1.60 $i\hbar$	0.17	1.00	-0.40
3.00	1.80	-(0.10 + 1.80 $i\hbar$)	0.15	0.60	0.10 - 1.80 $i\hbar$	0.17	1.00	-0.20
3.20	2.00	-(0.00 + 2.00 $i\hbar$)			0.00 - 2.00 $i\hbar$	0.17	1.00	0.00

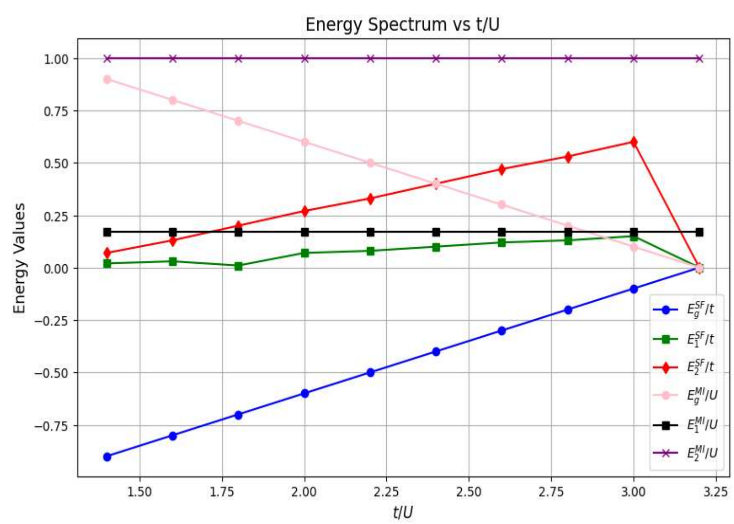

Fig. 3. Energy spectrum for the 3 bosons interaction with single particle hopping in the Superfluid (SF) and Mott-insulator (MI) phases at varying values of U/t and of t/U with repulsive vorticity. The respective plots are: (a) blue line for the SF ground state (b) green line for the SF first excited energy (c) red line for second SF excited energy (d) pink line for MI ground state (e) black line for MI first excited energy (f) purple line for MI second excited energy.

Table 5. Energy spectrum profiles of SF and MI phases for the 3 interacting bosons with pair hopping

U/t t t/U γ $(\times i\hbar)$	γ/t $(\times i\hbar)$ γ/U $(\times i\hbar)$	SF Energy spectrum E_g^{SF}/t	 E_1^{SF}/t	 E_2^{SF}/t	MI Energy spectrum E_g^{MI}/U	 E_1^{MI}/U	 E_2^{MI}/U	SF-MI transition ΔE_g
1.40	0.20	0.02	-0.10 $i\hbar$	0.10	0.17	0.90 - 0.20 $i\hbar$	1.00	-0.15
1.60	0.40	0.03	-0.20 $i\hbar$	0.20	0.17	0.80 - 0.40 $i\hbar$	1.00	-0.13
1.80	0.60	0.01	-0.30 $i\hbar$	0.30	0.17	0.70 - 0.60 $i\hbar$	1.00	-0.16
2.00	0.80	0.07	-0.40 $i\hbar$	0.40	0.17	0.60 - 0.80 $i\hbar$	1.00	-0.10
2.20	1.00	0.08	-0.50 $i\hbar$	0.50	0.17	0.50 - 1.00 $i\hbar$	1.00	-0.08
2.40	1.20	0.10	-0.60 $i\hbar$	0.60	0.17	0.40 - 1.20 $i\hbar$	1.00	-0.07
2.60	1.40	0.12	-0.70 $i\hbar$	0.70	0.17	0.30 - 1.40 $i\hbar$	1.00	-0.05
2.80	1.60	0.13	-0.80 $i\hbar$	0.80	0.17	0.20 - 1.60 $i\hbar$	1.00	-0.03
3.00	1.80	0.15	-0.90 $i\hbar$	0.90	0.17	0.10 - 1.80 $i\hbar$	1.00	-0.02
3.20	2.00		-1.00 $i\hbar$	1.00	0.17	0.00 - 2.00 $i\hbar$	1.00	0.00

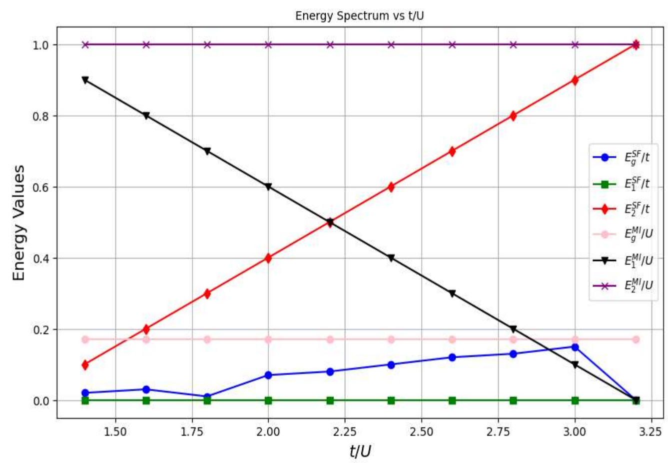

Fig. 4. Energy spectrum for the 3 bosons interaction with pair hopping in the Superfluid (SF) and Mott-insulator (MI) phases at varying values of U/t and of t/U with repulsive vorticity

The respective plots are: (a) blue line for the SF ground state (b) green line for the SF first excited energy (c) red line for second SF excited energy (d) pink line for MI ground state (e) black line for MI first excited energy (f) purple line for MI second excited energy.

3.2 DISCUSSION

In Table3 above, the critical value of U/t and t/U for the Superfluid to Mott-insulator transition is 2.00, in agreement with Manuela et. al.(2004) where the critical value is $U/t \approx 2.10 \pm 0.1$ in 1D Mott-insulator transition and also as calculated in Batrouni et. al. (1990). The energy spectra for the Superfluid phase and Mott-insulator phase in Equ.2.5 and Equ.2.7, respectively show positive ground state and first excited states, and are non-degenerate. These are in agreement with Feynman's no node theorem. From Fig.2, the Superfluid ground state energy is lower due to the delocalization of the bosons, while the Mott insulator's is higher due to the localization of the interactions. The absence of the imaginary parameter in the energy spectrum for the two bosons can be understood as non- effect of the vortex potential. Qualitatively, the absence of the vortex potential here mean the vortex might have escaped from the trap due to quantum tunneling. The existence of higher energy excitation in the Mott Insulator phase Kunher and Moniem (1998) as observed from our graphs on Fig.2 for the ground states at $t/U = 1.40, 1.60, 1.80$ and Figures.3 and 4 for all real parts , first excited state and second excited states. The first excited energy is double the ground state for both the SF and MI phases for the 2 bosons, with increasing change in energy as U/t increases. The values of U/t below the critical point, the interaction prefer to bring in a vortex ($\Delta E < 0$), while above the transition point the interaction prefer to remain without vortex at critical rotation, in agreement with the Fonda et al.(2014). It becomes energetically costly to tunnel between sites in the Superfluid phase due to the repulsive on-site interaction, such that the atoms cannot tunnel anymore and fail to maintain long range coherence in the Mott- insulator phase where the tunneling is frozen and each lattice site has a fixed number of bosons. We have taken this into cognizance being that for the ground state energies for the 2 bosons interaction is dominant above the transition point for both the SF and the MI phases, hence suppressing the effect of vortex.

In the regime of the 3 bosons interaction with single particle hopping and the pair hopping, the transition point as observed from Tables 4 and 5, including Figures 3 and 4 can be seen to be U_c/t and $t_c/U = 3.20$. This is closely equal to the SF to MI transition point $U_c/t \approx 3.40$ obtained by Jaksch et al. (1998), Kunher and Moniem (1998), Pai and Pandit (2005). For the single particle hopping process, the ground state is stable due to oscillations as interpreted by the equal imaginary values due to vortex, and the MI phase is stable at the first and second excited energies for all parameter values.

The SF phase at the pair hopping case from Table 5 is purely imaginary at ground state and the imaginary values for the MI phase at first excited state doubles its SF phase counterpart, showing high vorticity. The negative values for the change in the ground state energies for all values in the parameter range signifies that the 3 bosons interactions would prefer to bring in vortex in the dynamics of the particles. The degeneracy in the three bosons interaction is an intriguing quantum effect. As in quantum mechanical system, the degenerate ground states show some spontaneous symmetry breaking, which can be as a result of vortex nucleation. This seem to contradict the Feynman's no node theorem in the paper Wu (2009). In the Wu's explanation, the ground state of a bosonic system is positive real and therefore non-degenerate. However, the degenerate ground states for the three bosons interaction imply a violation of the symmetry of the Bose- Hubbard Hamiltonian. The existence of degenerate states arises from the underlying symmetry of the system. Due to rotational dynamics, the presence of vortex can break certain symmetries while preserving others, leading to multiple excited states that degenerate. The presence of higher excitation states is due to the increase in the number of bosons in the system which leads to more symmetry in the Bose- Hubbard Hamiltonian which induces more degeneracy in the energy spectrum for the pair hopping.

CONCLUSION

We have probed into the mechanism of vortex in Superfluid-Mott insulator transition by introducing the Bose-Hubbard mode considering neutral bosons confined in a 1D confinement which rotates about the x-axis in the x-direction. Our simulation results show that the responses of the system to the vortex depends on the number of bosons interacting in the system and significantly dependent on the nature of the hopping process. While the 2 bosons interaction suppresses the effect of vortex due to escape of the vortex from the trap as the particles tunnel the sites, for the three bosons the energy spectra exhibit vortex induced oscillations as interpreted by the equal imaginary values for some parameter values. The existence of degenerate states arises from the underlying symmetry of the system. Due to rotational dynamics, the presence of vortex can break certain symmetries while preserving others, leading to multiple excited states that degenerate. The presence of higher excitation states is due to the increase in the number of bosons in the system which leads to more symmetry in the Bose- Hubbard Hamiltonian which induces more degeneracy in the energy spectrum. The effect of vortex on the 1D Superfluid to Mott insulation has no significant effect on the transition points, but has much significant effect on the energy optimization of the system, and the degeneracy can influence the system's thermodynamics properties, and lead to collective excitation. Our simulation result is a good candidate for implementation of the Dirac's principle of superposition of evolution of time Ibeh and Akpojotor (2020) in probing the problem of "negative time" in quantum mechanics.

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